Monetary Policy with Interest on Reserves

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1 Introduction

During the last few years, Federal Reserve policy has made many enormous changes. I focus on two: First, the Fed now pays interest on reserves (accounts that banks hold at the Fed), which used not to pay interest. Second, the Fed has amassed an unprecedented balance sheet. Before the crisis, the Fed held about a trillion dollars of US government bonds, corresponding to about $950 billion dollars of currency and $50 billion dollars of bank reserves. Through its various quantitative easing programs, the Federal Reserve has accumulated another $2.5 Trillion of assets, and created about $2.5 trillion of additional reserves in exchange. The Fed bought US treasury bonds and mortgage-backed securities, and created interest-paying reserves in exchange. (We could say "printed money," but it's easier than that. The Fed creates reserves by simply programming a computer.) These are only two of the huge number of policy innovations, but they are fundamental enough that analyzing just these two will occupy us for a whole essay, and then some.

Soon, if it has not happened already by the time this article is published, interest rates will start rising again. When that time happens, the Federal Reserve faces a big choice: Will it, or must it, go back to "normal," a small (now about $80 billion) quantity of reserves that do not pay interest, which will require the Fed to sell off or somehow otherwise soak back up $2.5 trillion of reserves? Or will it leave in place the huge balance sheet, and either cause or accommodate (it's hard to tell which sometimes) a rise in interest rates by simply paying higher rates on reserves?

My reading of Federal Reserve statements, starting with Chairman Bernanke's (2010) testimony, is the latter course. It is a course I endorse wholeheartedly. A huge balance sheet, with reserves that pay market interest, is a very desirable configuration of monetary affairs.

However, interest on reserves, together with the spread of interest-paying electronic money, radically changes just about everything in conventional monetary policy analysis. Standard answers to fundamental questions like the determination of inflation, the ability of the Fed to control real and nominal interest rates, the channels of the effect of monetary policy especially on the banking system, and so forth all change dramatically in a regime of interest on reserves and large balance sheet. The Fed anticipates some, but not others. Old habits die hard, and clear thinking is needed to dispel them. In addition, the presence of a large stock of outstanding debt, most relatively short

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maturity, means that interest rate changes will have much deeper fiscal impacts than heretofore, a limit on monetary policy that the Federal Reserve is barely thinking about at all.

1.1 Why interest on reserves is a good thing

Why does it matter so much whether the Fed pays interest on reserves with a large balance sheet, or reverts to a traditional very small balance sheet with non-interest-paying reserves? The short answers: optimal quantity of money, and financial stability.

Optimal quantity of money

Milton Friedman (1969) described the "optimal quantity of money:" a slow deflation with zero nominal rates and deflation generating the a positive real rate of interest. In this environment, money and bonds pay the same rate of return. Then, the liquidity premium for money goes to zero, "shoe leather" costs of cash management go to zero, and the economy is satiated in liquidity. Since there is no social cost to this liquidity, it's the optimal quantity of money.

The same circumstance arises if we have interest-paying money, that pays the same interest as bonds. In this case, again, people face no artificial incentive to economize on money balances, the economy is satiated in liquidity, and shoe-leather costs go to zero. Lucas (2000) estimates the welfare benefits of going to the optimal quantity of money, the area under the money demand curve, at around 1% of GDP, a not inconsiderable sum.

People have objected to Friedman's proposal, that prices and wages might be sticky so perpetual deflation might not be a good thing. Moreover, living at the zero lower bound means the Fed does not have the power to lower real rates in response to various shocks. But if we had interest-paying money, then we could have all the benefits of the optimal quantity of money at zero (my preference) or positive (the Fed's preference) inflation.

For this reason alone, economists for centuries have dreamed of interest-paying money.

Until the last decade or so, however, interest-paying money was impractical. We could not track down the holder of a dollar bill and pay up his interest, and the holder of dollar bill could not know exactly how much interest his bill had accrued.

But instant communications, instant computation, and electronic payments have changed all that. Now, the vast majority of the legal economy has moved to interest-paying electronic money. Almost all of us pay by credit cards or debit cards, linked to accounts that will, when interest rates rise, pay interest. Business moved to interest-paying electronic money a while ago.

Cash still exists in rather surprising quantity – about a trillion dollars, or more than $3,000 per capita, almost all in the form of hundred dollar bills. With traces of cocaine on them. Cash really is only used in any substantial quantity for illegal transactions, undocumented people, and economics textbooks, which are soon becoming history of economics textbooks. Most importantly for our purposes, any quick glance at the data shows that cash just trundles along and is disconnected from the economy and, especially, the financial system.

For the rest of this essay, I will simply ignore cash, and think of a monetary system based on reserves – electronic accounts that banks hold with the Federal reserve. Private debts and government debts ultimately promise reserves. Anything settled by a bank is such a promise. Though most transactions are in fact netted between banks, the net difference is still settled with reserves. This little abstraction does very little violence to the functioning of the US financial economic and monetary system, for which cash and coins are really irrelevant.

Financial stability

However, our interest-paying money was all inside money, circulating promises to deliver reserves: Interest-bearing checking accounts and deposit accounts, money-market funds, overnight repurchase agreements, uncollateralized overnight lending, short-term commercial paper, auction-rate securities, and so forth. We were holding something like $10 trillion of promises to pay the then roughly $50 billion dollars of reserves, an enormous multiplier.

These promises were backed by assets. If people wanted to be paid, each issuer planned to sell or borrow against assets such as mortgage-backed securities to raise the needed cash. But asset values can fall, and issuers can fail. Worse, when people fear failures, everyone demands payment at the same time. The issuers cannot collectively fulfill all their promises, and the financial system fails. Interest-paying inside money can suffer the same systemic run as non-interest paying banknotes can suffer.

Now, unlike in the gold standard era, we have a more elastic supply. In our crisis, the Fed tried to help, buying or lending against many of the dodgy assets that banks and shadow-banks were trying to sell. But there are limits to what the Fed can do. So, we had a classic systemic run; a failure of fractional-reserve shadow-banking.

Narrow banking is the natural cure: we can hold reserves directly, we can hold similar fixed-value floating-rate electronically transferable Treasury debt, or we hold interest-paying money issued by intermediaries such as banks or money market funds that are 100% backed by interest paying reserves or very short-term treasury debt. (Another technological change: There is no real reason that the central bank must transform the liquidity of government debt, as the treasury can itself issue debt as liquid as reserves, or even more so, since only banks can hold reserves and anyone can hold treasury debt.) Fractional-reserve banking cannot collectively deliver all the promised money, unless the Fed agrees to buy every asset used as backing in a crisis, transforming inside money into outside money. If inside money depends on the right to convert the whole stock to outside money in a crisis, we might as well start there in the first place.

As the Federal Government displaced private note issue in the 19th century, so it can displace private interest-paying money in the 21st, with similar benefits to financial stability. The government does a few things well, and has a few national monopolies. Money is one of them, and now interest-paying electronic money. Fiat currency is not without its occasional inflations and crises either. But ultimately, the government can always avoid explicit default by printing money – government debt only promises more government debt. Ultimately, government debt is valued because you need maturing government debt to pay taxes, so government debt is backed by the present value of the government's ability and willingness to soak up money by taxing its citizens in excess of spending. That backing and ability to avoid default is better than backing by assets such as houses, and a plan to avoid default by selling assets.

That's all a long way away. But in a first step, before we get grandiose and think about *forcing* narrow banking through regulation, before we think about regulating away the shadow-banking money-provision that failed, we can at least put a heavy emphasis on *allowing* narrow banking, and letting government-provided interest-bearing money compete and contribute to financial stability.

So this is fundamentally why a large balance sheet and interest-paying reserves are beneficial. A Federal Reserve with $2.6 Trillion of interest-paying excess reserves outstanding is a financial system with $2.6 trillion of narrow-banking deposits outstanding, a great advance in financial stability!

Kashyap and Stein (2012) advocate something similar: using interest on reserves to "manage the inflation-output tradeoff" and balance sheet and reserve requirement control to "regulate the externalities created by socially excessive short-term debt issuance on the part of financial intermediaries." Kashyap and Stein want to ration reserves and broaden reserve requirements, and use the spread on reserves vs. treasuries as a macroprudential policy tool. Rather than actively manipulate these externalities, I regard paying full interest on reserves as the first step to eliminating them by eliminating any substantial short-term debt issuance. So, we both want to emphasize financial stability, but I think it comes from narrowing the spread between reserves and treasuries with a large balance sheet, and they think it comes from widening that spread, controlling quantities, and a controlled balance sheet.

1.2 Interest-paying money

The shift to interest-paying money dramatically changes how money works, how monetary policy works, and how inflation is determined. This change is the same whether the system is based on a small amount of interest paying reserves and a large amount of interest-paying inside money, judiciously regulated by our officials to avoid future crises, or whether based on a large amount of interest-paying reserves and 100% reserve banking.

Let us abstract the financial system as follows. Again, ignore cash. At the foundation is short-term government debt. Ideally, the Treasury would directly issue fixed-value floating-rate electronically transferable debt, but if not, the Federal reserve buys short-term government debt and issues fixed-value floating-rate electronically transferable debt called bank reserves. Money market funds provide the same service directly. Private short-term debts promise delivery of one of these two forms of government debt. The government accepts reserves or its own fixed value floating-rate debt in exchange for tax payments, and delivers such debt as expenditures. All overnight government debt pays the same rate, so reserves and overnight treasury debt pay the same rate. (I will think below about Fed plans to separately control the balance sheet and to try to pay less on reserves than Treasuries. I will argue against the idea.) Transactions can be settled with electronic delivery of reserves or corresponding treasury debt.

One can think of this system as interest-paying electronic money. One can also think of this system as an economy with no money at all. The electronic revolution allows us to transfer assets based on government debt, which we hold for savings purposes, in order to settle transactions. One can think of this system as a simple electronic barter economy.

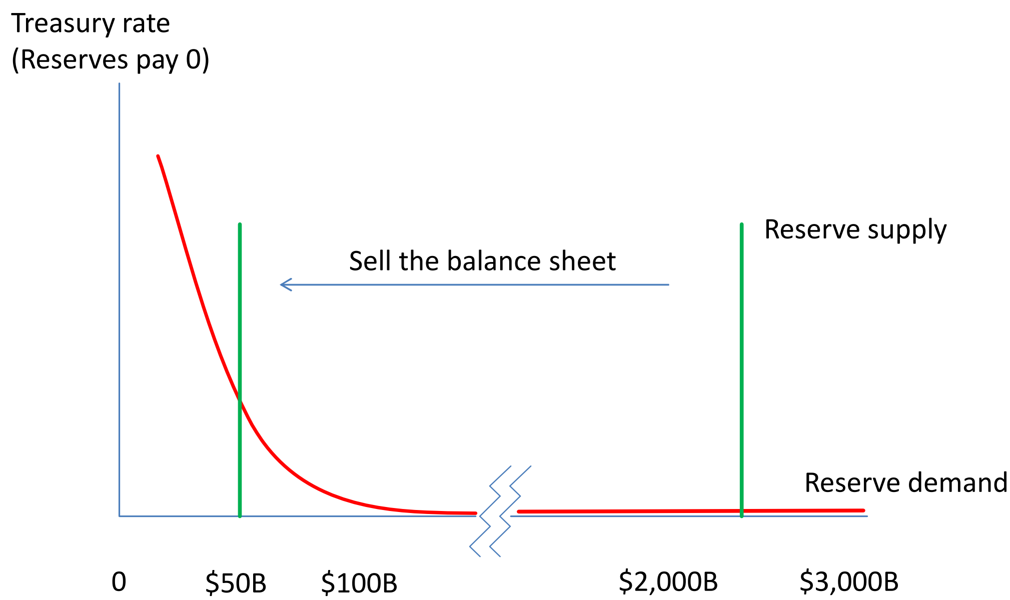
In fact, the electronic revolution means that we don't really need an inventory of *fixed-value* assets to make transactions either. You could quite easily buy a cup of coffee by transferring a share of an S&P500 index fund. This was not possible historically, because of spatial separation: you and the barista don't know what the value of the S&P500 index is. But now you do, in milliseconds. Even if the transaction is, by convention, mediated by the transfer of short-term government debt, you could sell your fund share in 20 milliseconds, transfer the government debt in another 20 milliseconds, and the barista could buy a share of a mortgage-backed security ETF in another 20 milliseconds, all with transactions costs far less than today's standard 4% credit card fee. The inventory of fixed-value debt the economy needs to facilitate this transaction, with you and the barista holding it for 20 milliseconds, is infinitesimal. Moreover, this transaction would undoubtedly be netted in practice – if you share the same bank, it simply changes your book entry and the barista's book entry.

1.3 A different mechanism

The interest-on-reserves mechanism is radically different from the standard story we tell for how the Fed controls interest rates, and how Fed actions influence the banking system, the real economy, and ultimately inflation. (I use the word "story" deliberately as I think we have in fact been much closer to the interest on reserves mechanism already than is commonly realized.)

Consider the standard story for the standard mechanism, if the Fed wanted to tighten, what would it do, and how would it work?Figure 1.3 illustrates. In the standard story, the Fed controls interest rates by rationing the amount of non-interest-paying reserves. Banks must hold reserves in proportion to their deposits. If the Fed sells bonds, taking back reserves, the banks must get along with fewer reserves. They bid up the price that they pay to borrow reserves from each other – the Federal Funds rate. They sell short - term bonds to try to raise reserves, an effort that may be individually successful but cannot collectively do anything but raise the short-term treasury rate. So the Federal funds and Treasury rates rise.

But, required reserves were about $50 billion before the financial crisis. They are now about $80 billion, reflecting the growth of inside money at zero rates, but much of that money pays interest, so the figure may decline somewhat. So, in order to tighten, the Fed would have to sell off almost its entire $2.6 Trillion balance sheet expansion.



In turn, the story goes, banks with fewer reserves must lend less and undo deposit creation. Thus the open market operation cut in the size of the balance sheet, through the money multiplier, forces a cut in lending and a cut in money held by the public in the form of demand deposits. Finally, in this story, the price level is set by money, both cash and bank deposits, via MV=PY and some long and variable lags. Eventually, the price level falls.

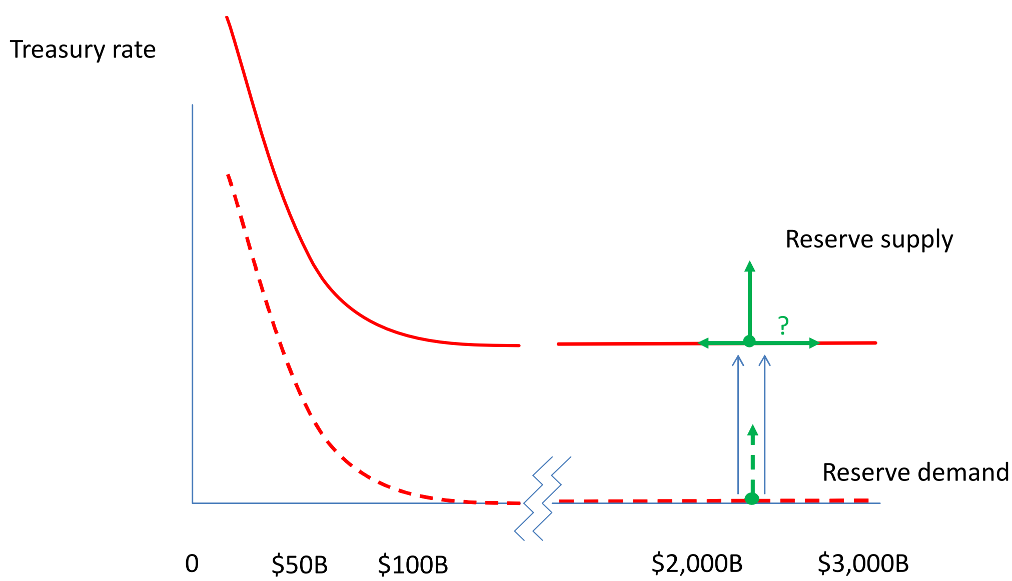
Now, consider the interest-on-reserves channel. I attempt a parallel graph in Figure 1.3. The Fed doesn't sell anything, but simply raises the interest rate it pays on reserves. I graph this by moving the point, corresponding to the current reserve supply and interest rate, *up*, not to the left.

The first question: Can the Fed even control interest rates? If so, how? The *mechanism* for interest rate control is completely different. If you said in the past, in response to such questions, "Sure, the Fed controls interest rates. It rations the supply of money, and then we work down the money demand curve," that reply is *completely* irrelevant now. We remain in the region where money demand is undefined. In the usual presentation with the interest rate spread between treasuries and money on the horizontal axis, and quantity on the vertical axis, we remain exactly at the same point, where the spread is zero and the quantity is indeterminate.

New-Keynesian models such as Woodford (2005) specify interest rate control with no money. But as Woodford makes clear, such models consider a limit. Woodford still relies on the traditional mechanism for interest rate control; he just advocates the sensible limit that reserves ($50 billion) are so small as to be effectively zero.

Sure, the Fed can announce an interest rate on reserves, and can pay the interest by simply printing up new reserves when the time comes. But just how will that rate spread to all other interest rates? Can the Fed change all rates without losing control of its balance sheet, as indicated by the vertical green line? Or, to bring up Treasury rates, must the Fed consent to any size balance sheet at the desired rate, "give us your treasuries, in any quantity, we will give you 5% reserves," illustrated by the flat curve?

Next, assume that the interest rate on Treasuries, deposits, loans, and so forth do move, to equal the interest rate on reserves, as desired. In that case, the reserve demand curve moves up as well. (The "demand curve" is really a function of the spread between reserves and treasuries. Read it as, "what must the equilibrium rate on Treasuries be so that banks are willing to hold the price and quantity of supplied reserves?")



In the new environment, however, *banks remain satiated in reserves*, and the economy remains satiated in money.

This fact means that the entire standard transmission mechanism fails completely. Once, we would say that if a bank wanted to make a loan, it created a deposit out of thin air. But such deposit creation was limited by the quantity of reserves. Hence, open market operations – changes in the size of the balance sheet – controlled bank lending and the money supply. But now, with abundant excess reserves, banks can create loans and deposits at will. They might be limited by capital requirements or regulation (a temptation the Fed will have to fight, another issue I take up below), but not at all by reserves. Conversely, changes in reserves supply or the size of the balance sheet have no effects at all on lending or inside money creation. The money multiplier is zero; deposits and loans are unhinged from reserves. The mechanism, if one exists which connects monetary policy to the economy is entirely different.

The Fed already pretty much recognizes this fact. In describing how quantitative easing – open market operations with zero spread between reserves and treasuries – works, the Fed has emphasized the effects on *bond* supply, not *money* supply. Again, this is a radical change and a complete reversal. Historically – at least since Friedman (1968) – the effects of monetary policy have been supposed to come from changes in the supply of money which is issued, not changes in the supply of bonds which are bought. MV=PY states that it does not matter what assets are bought, or if any assets are bought at all – MV = PY states the equivalence of an open market operation to a helicopter drop.

Which leads us to the most fundamental question of all: How does monetary policy now affect inflation? Open market operations are disconnected from deposits, so we cannot pretend that the supply of money is controlled at all. And if money pays the same interest as bonds, the demand for money is indeterminate. MV=PY now is a definition of V, since the composition of private sector assets between M (reserves) and treasuries is irrelevant.

Will inflation become uncontrolled in an interest on money regime, awash with liquidity? Already a number of authors in the monetarist tradition have advocated limiting financial innovation so that MV=PY can be brought back again and the price level controlled. But that cat is out of the bag.

Then, how do interest rate changes on reserves feed through to inflation? Even if you regard these as empirical questions, all of our historical experience derives from one regime, with rationed liquidity, an operating money demand curve, binding reserve requirements, and so forth. It is not obvious that the same impulse-response functions will characterize such a fundamentally different causal mechanisms.

2 Inflation and nominal interest rate targets

I start with the simplest possible model, to answer the most fundamental questions: Can the Fed control nominal interest rates, and will inflation be determined in the interest on money regime? I answer both questions affirmatively. I use a completely frictionless model – neither monetary frictions nor pricing frictions. The absence of monetary frictions is crucial – the whole innovation of interest-paying money and interest on reserves is that there will be no monetary frictions. So, let us start by reexamining what monetary policy can do without monetary frictions. The absence of pricing frictions lets us start by thinking of a standard benchmark, and gives some sense of standard “long run neutrality” predictions. I add pricing frictions below.

I base this analysis on the valuation formula for government debt,

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}. \tag{1}$$

Here, B_{t-1} is the nominal value of government debt outstanding at the beginning of time t, P_t is the price level, $\beta = 1/(1+r)$ is a constant real interest rate, and s_t are real primary surpluses.

This formula, often made more complex with various frictions, is part of every well-specified model. By starting with a very simple model in which this is the only remaining formula involving nominal quantities, we obtain very general assurance about what monetary policy can and cannot do, not a theory that is special in some way.

We need to base our analysis on some view of price-level determination, and the fiscal backing expressed by equation (1) really is the only hope for analyzing the interest-on-reserves regime. All the familiar stories for inflation determination that ignore fiscal backing fall apart in the interest on reserves regime. Money might be valued because it is scarce; the quantity theory; MV=PY. But under the interest on reserves regime, the whole point is that money is not scarce. We will be satiated in liquidity. And already, interest rate targets do not limit money supply. Money might be valued, or at least inflation determined because the Fed controls interest rates. But Sargent and Wallace (1981) taught us that though an interest rate target may control expected inflation, $i_t = r + E_t\pi_{t+1}$, it cannot determine unexpected inflation $\pi_{t+1} - E_t\pi_{t+1}$. The attempt to restore determinacy via a Taylor-type rule in a new-Keynesian model, adding $i_t = \phi\pi_t, \phi > 1$, also fails, as documented at great (!) length in Cochrane (2011). These are ultimately backing-free theories of inflation, and require some monetary friction. In the absence of a monetary friction, the only way to value money is via its backing. Fortunately, our fiat money is backed: the government accepts its money, and only its money, in payment of taxes. This backing by the present value of future taxation is in fact a much better backing than gold reserves or other potential backing for liquid assets. And both MV=PY and interest rate theories *do* include a fiscal backing. It's just hidden in footnotes, and needs to be brought back to the center of events.

2.1 A digression on the valuation formula

Use of (1) to think about inflation is clouded in myriad unnecessary controversies, which are worth clearing up for readers uncomfortable with that analysis.

It is helpful to derive (1) in a fully-specified model, which I do in the Appendix. The representative consumer maximizes $E\sum_t \beta^t u(c_t)$ and has a constant endowment y. This specification produces a constant real interest rate $1 + r = 1/\beta$.

The government sells one-period nominal debt with face value B_{t-1} at the end of time $t-1$. It redeems debt with money at the beginning of time t, then soaks up that money at the end of time t with lump-sum real surpluses s_t and bond sales with value Q_tB_t, where Q_t is the one-period bond price. Interest is paid overnight, and people do not want to hold money overnight, so money printed in the morning must be soaked up in the afternoon,

$$B_{t-1} = P_t s_t + \beta E_t\left(\frac{P_t}{P_{t+1}}\right)B_t$$

or, in real terms.

$$\frac{B_{t-1}}{P_t} = s_t + \beta E_t\left(\frac{B_t}{P_{t+1}}\right) \tag{2}$$

Iterating forward and applying the consumer's transversality condition, we obtain the basic equilibrium condition (1)

Equation (1) is not a "budget constraint." It is a valuation equation, an equilibrium condition. It works the same was as the valuation equation by which stock prices adjust the present value of expected dividends. There is no "budget constraint" that forces the government to respond to a deflation in P_t by raising surpluses arbitrarily high, any more than a stock price "bubble" forces a company to raise earnings to justify the stock price.

Equation (1) has a natural "aggregate demand" interpretation. (Woodford 1995). If the real value of nominal debt is less than the present value of surpluses, then people try to spend their money on goods and services. But collectively, they can't, so this "excess aggregate demand" just pushes up prices until the real value of debt is again equal to the present value of surpluses. Aggregate demand is nothing more or less than demand for government debt, as by the private-sector budget constraint the only way to spend more on everything else is to spend less on government debt. This equation also expresses a "wealth effect" of government debt.

Though the literature spends a lot of time thinking about "regimes" and testing for them, there is really not much point to that exercise. Equation (1) or a variant holds in every well-specified model. If the treasury sets surpluses $\{s_t\}$ following some other signal sent by the Fed, nonetheless it is the value of surpluses which, through aggregate demand, cause inflation or deflation.

As a simple example, suppose we modify the model to add a demand

$$M_t V = P_t y \tag{3}$$

for money held overnight. (1) now is modified to include a seignorage term,

$$\frac{B_{t-1}}{P_t} = \sum_{j=0}^{\infty} \beta^j E_t \left[s_{t+j} + \frac{M_{t+j} - M_{t+j-1}}{P_{t+j}} \right]. \tag{4}$$

or equivalently

$$\frac{B_{t-1} + M_{t-1}}{P_t} = \sum E_t \left[m_{t,t+j} \left(s_{t+j} + \frac{i_{t+j}}{1 + i_{t+j}} \frac{M_{t+j}}{P_{t+j}} \right) \right]$$

Both equations must hold in equilibrium.

Now, following Leeper (1991) we often talk of a money-dominant "regime" as one in which the Fed sets M_t, P_t follows from (3), and then the Treasury sets $\{s_t\}$ in (4) to validate the Fed-chosen P_t, and a fiscal-dominant "regime" as the opposite case. But both equations hold in both regimes, so there is no testable content to the regime specification from observations of $\{M_t, B_t, P_t, s_t\}$, (Cochrane 2011a) which should already alert us to the sterility of the investigation. Is it the foot on the gas pedal, or the engine which ultimately causes the car to go? If a man (Fed) induces a horse (Treasury) to pull a cart by putting a carrot under the horse's nose, does that mean the man pulls the cart?

The same point holds if the Fed is imagined to follow an interest rate target with a Taylor rule. Again, a version of (1) holds, and the Treasury is assumed to adjust $\{s_t\}$ to validate the model's price-level predictions; if the Treasury will not or cannot, the hypothesized price level won't form. There is no testable content to whether the Treasury or Fed drives the "regime," and we might as well interpret the fiscal backing as the ultimate cause of inflation.

Money and fiscal policy must always be coordinated. One can view a money-dominant regime as one in which the Fed's M_t choice merely prods the Treasury to make the necessary s_t adjustment, or even exists only to communicate the Treasury's $\{s_t\}$ intentions, and the latter, through (4) is the ultimate "causal" force for inflation or deflation. "Aggregate demand" must in the end add up to demand for all all government debt. Monetary contractions without fiscal support and coordination fail. Fiscal contractions with loose money stop inflations (Sargent and Wallace 1981). If the Fed were to try a 50% deflation now, this would mean doubling the real value of publicly-held debt from \$12 trillion to \$24 trillion, and the value of the government's credit guarantees by more than that. A call to the Treasury to double taxes would quickly discover the top of the Laffer curve in the rear-view mirror.

As these examples emphasize, for (1) to hold and play a central role in price determination, one does not have to, and one should not, think of surpluses $\{s_t\}$ as being "exogenous," or set without regard to other variables, including prices. Equation (1) tells us what the equilibrium price level must be, conditioned on the equilibrium $\{B_t\}$ and $\{s_t\}$. That is all.

Likewise, it's tempting and useful for comparative-statics exercises to think about fixing $\{s_t\}$ or $\{B_t\}$ holding the others constant. However, real monetary and fiscal policy is always coordinated, and most events contain large movements in both quantities at the same time. For example, wars and recessions feature big increases in debt B_t with big negative current surpluses s_t. But these events come with big increases in expected future surpluses $E_t s_{t+j}$, because governments want to raise real revenue, not cause inflation. So $\{s_t\}$ follows a response that is negative now, and positive later, to such a shock, nothing like an AR(1), and $\{s_t\}$ and $\{B_t\}$ move together in response to such typical economic shocks.

2.2 Monetary policy and inflation in the simple model

Having established, I hope, that equation (1) is a constituent of all models rather than a specific model, in which all the usual frictions have been stripped out, and thus showing us the most general results rather than particular ones, we are ready to use (1) to think about inflation determination in the interest on reserves regime.

Examine the expected and unexpected components of (1), which I repeat,

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j},$$

namely,

$$\frac{B_{t-1}}{P_{t-1}} (E_t - E_{t-1}) \left(\frac{P_{t-1}}{P_t} \right) = (E_t - E_{t-1}) \sum_{j=0}^{\infty} \beta^j s_{t+j}. \quad (5)$$

$$\frac{B_{t-1}}{P_{t-1}} E_{t-1} \left(\frac{P_{t-1}}{P_t} \right) = E_{t-1} \sum_{j=0}^{\infty} \beta^j s_{t+j}. \quad (6)$$

Equation (5) shows us that *unexpected inflation is determined entirely by fiscal policy*, where by "fiscal policy" I mean changes in current and expected surpluses $\{s_t\}$.

By contrast, Equation (6) shows that *the government can entirely determine expected inflation by nominal bond sales B_{t-1}, even with no change at all in fiscal policy,* $\{E_{t-1} s_{t+j}\}$. This is the central result, which we need to digest and interpret.

To clarify the effect, note that the one period bond price is

$$Q_{t-1} = \beta E_{t-1} \left(\frac{P_{t-1}}{P_t} \right) \quad (7)$$

and therefore, (6) reads

$$\frac{B_{t-1} Q_{t-1}}{P_{t-1}} = E_{t-1} \sum_{j=0}^{\infty} \beta^{j+1} s_t \quad (8)$$

P_{t-1} is already determined, so the real value that the government raises by debt sales $B_{t-1}Q_{t-1}$ is fixed by the present value of real surpluses. Thus, if the government sells more B_{t-1}, it faces a unit-elastic demand curve; the nominal bond price Q_{t-1} falls one for one, because inflation $E_{t-1}(P_{t-1}/P_t)$ rises one for one. This is just like a share split or a currency revaluation. But like those examples, it shows that the government has complete power over units in this frictionless model, which means it can control expected inflation without changing anything real.

Furthermore, writing (8) as

$$\frac{B_{t-1}}{P_{t-1}}\frac{1}{1+i_{t-1}} = E_{t-1}\sum_{j=0}^{\infty}\beta^{j+1}s_{t+j} \tag{9}$$

we can transform a government decision of the quantity of debt to sell B_{t-1} to a much more attractive interest rate target.

In sum, then, *in this frictionless model, the government can set a nominal interest rate target; it can set that target without any adjustments to fiscal policy* $\{E_t s_{t+j}\}$*, and by setting nominal interest rates, the government can control expected inflation.* To set an interest rate target, the government auctions bonds – it says "the nominal interest rate is 5%, and we are selling nominal bonds at 1/0.95 dollars per face value. We will sell any amount demanded at that price." Equation (9) now is a simple reading of private demand – if the government targets nominal interest rates at a level i, how many bonds B_{t-1} will be demanded.

The combination (6) and (5) extend old issues in monetary economics. The Fisher relationship $i_{t-1} = r + E_{t-1}\pi_t$ already says that by controlling nominal interest rates, the government could control expected inflation. But it is not clear by that simple statement just *how* the government could control nominal interest rates, and that control left open the Sargent-Wallace (1981) indeterminacy result, that $\pi_t - E_{t-1}\pi_t$ could be anything. Now we see that the government *can* set the nominal interest rate, and the action it must do to achieve that result: auction nominal bonds with fixed surpluses. W*e also see that the government valuation equation solves the indeterminacy problem.* Ex-post inflation and deflation are pinned down by the valuation of government debt. Even a completely fixed interest rate target does not lead to inflation instability or indeterminacy. A Taylor-type rule reacting to inflation is not needed.

2.3 Mapping the simple model to our world

Now, let us step back a bit to apply this model to monetary policy and current institutions a bit better. It's pretty straightforward to think of $\{s_t\}$ choices as "fiscal policy."

In this model, there is no difference at all between interest-paying reserves and Treasury debt held directly by the public. The symbol B_{t-1} refers to the sum of the two quantities. So without further frictions, "monetary policy" of open-market purchases of one-period debt in exchange for interest-paying reserves has no effect at all.

But if we understand "monetary policy" as "changes in the quantity and structure of government debt with no change in taxes or spending," then monetary policy is in fact quite effective here. And this is a good definition. The heart of central banking is that central banks are forbidden to undertake direct fiscal actions. For example, despite many commenter's desire for the opposite, central banks may not undertake helicopter drops. Helicopter drops, writing checks to voters, are fiscal (plus monetary) policy, and change $\{s_t\}$.

This model then states that the Fed *can*, by announcing an interest rate on reserves affect all nominal interest rates, and that the Fed *can* by so doing control expected inflation completely.

This model warns us though that the Fed may not be able to separately control the interest rate on reserves and the size of its balance sheet, which is undetermined here. Moreover, the government as a whole cannot separately control the nominal interest rate and the quantity of nominal debt that is issued. To set the nominal rate here, the government must sell whatever amount of nominal debt B_{t-1} the market demands at that rate. (Or, it must use (9) to successfully forecast that demand.) It cannot auction a fixed quantity.

In intuitive terms, when the Fed raises interest rates on reserves in this simple model, it must also agree to take on as much treasury debt as is offered – "Bring us your treasuries, we give you 5% reserves instead," and if it buys up the entire stock of treasury debt at that price, the Treasury must be willing to issue more debt for the Fed to buy.

The presence of expected surpluses in all these equations suggests a reason for institutional separation between Treasury and Fed. When the Treasury sells more debt it does not explicitly tie that debt to higher future surpluses, as many municipalities do. When the Treasury sells more debt, it *wants* to raise more real revenue, and it does not want to cause inflation. Increasing $\{B_{t-1}\}$ with no change in $\{s_t\}$ would raise no revenue and would cause inflation. The Treasury wants to communicate a simultaneous rise in promised real surpluses $\{E_{t-1}s_{t+j}\}$ with the new debt sale precisely to avoid these effects. By contrast, the "Fed" wants to communicate the opposite expectations in such sales. To control nominal rates, it wants to communicate that any changes in debt have no implications about future surpluses or changes in government revenue. Isolating the debt sales in two distinct branches of the government, one with no legal authority to do anything fiscal, is a great way to communicate different expectations of future surpluses with otherwise identical debt sales. In the same way, corporations market share splits – fully-diluting increases in shares outstanding with no changes in earnings – and public offerings – increases in shares outstanding that are intended to fully correspond to changes in earnings with no dilutions– in ways that convey the right expectations.

2.4 Long-term debt and quantitative easing

So far, I assumed that the government only issues one-period nominal debt. The U.S. maturity structure is in fact pretty short, with most debt rolling over in less than two years. So, we can apply these simple equations as a first approximation if we think of the "period" as at least two years. But thinking about long-term debt quite substantially changes the possibilities posed even by this very simple frictionless model. Cochrane (2001) undertakes a deeper analysis, but I can present a simple example here.

Suppose at time $t = 0$ the government issues two maturities of debt, overnight debt B_0^1 coming due at time $t = 1$, and B_0^{10} coming due at time $t = 10$. At time 1, also sell or repurchases some additional $t = 10$ debt. Let the amount of time $t = 10$ debt outstanding after the purchase and sale be B_1^{10}, so the purchase or sale is in the quantity $B_1^{10} - B_0^{10}$.

Now, the flow equation for time $t = 1$ is

$$B_0^1 = P_1 s_1 + Q_1^{10}\left(B_1^{10} - B_0^{10}\right)$$

where Q_1^{10} is the nominal bond price at time 1 for bonds that come due at time 10. Using the constant real rate,

$$\frac{B_0^1}{P_1} = s_1 + \frac{\left(B_1^{10} - B_0^{10}\right)}{(1+r)^9} E_t \left(\frac{1}{P_{10}}\right)$$

The flow equation at time $t = 10$ is simply

$$\frac{B_1^{10}}{P_{10}} = s_{10}.$$

Using that value to substitute,

$$\frac{B_0^1}{P_1} = s_1 + \frac{\left(B_1^{10} - B_0^{10}\right)}{B_1^{10}} \frac{E_1(s_{10})}{(1+r)^9}. \tag{10}$$

Again, take unexpected values to find

$$B_0^1 \left(E_1 - E_0\right) \left(\frac{1}{P_1}\right) = \left(E_1 - E_0\right) s_1 + \frac{1}{(1+r)^9} \left(E_1 - E_0\right) \left\{ \frac{\left(B_1^{10} - B_0^{10}\right)}{B_1^{10}} s_{10} \right\}.$$

Now, equation (10) offers an exciting new opportunity compared to equation (5): *by unexpectedly selling or repurchasing additional long-term debt, monetary policy can control unexpected inflation at time 1, without any fiscal policy change.* If we hold fixed the surpluses s_1 and s_{10}, then this equation reads

$$B_0^1 \left(E_1 - E_0\right) \left(\frac{1}{P_1}\right) = \frac{Es_{10}}{(1+r)^9} \left(E_1 - E_0\right) \left\{ \frac{\left(B_1^{10} - B_0^{10}\right)}{B_1^{10}} \right\}.$$

By unexpectedly selling more time - 10 debt, the government dilutes existing claims to time 10 surpluses. This action raises revenue (as debt sales did not in a one-period debt world), and that revenue can lower inflation at time 1.

There is a catch however: Selling additional long-term debt raises inflation at time 10,

$$\left(E_1 - E_0\right) \left(\frac{B_1^{10}}{P_{10}}\right) = Es_{10}.$$

Any increase $\left(E_1 - E_0\right) B_1^{10}$ without a change in expected surpluses shows up in a corresponding change in $\left(E_1 - E_0\right) (1/P_{10})$. In sum,

Monetary policy – changes in the maturity structure of government debt with no change in fiscal stance – can affect unexpected inflation. It does so by rearranging the path of inflation, delaying inflation or bringing inflation forward.

We may read current "quantitative easing" as exactly the opposite policy. By unexpectedly (relative to when the debt was sold) *buying* long-term debt, the Fed is trying to "stimulate," i.e. to *increase* inflation today, in exchange for less inflation later on.

Of course, monetary policy, fiscal policy, and real shocks in reality all coevolve. For example, a Fed that wanted to stabilize prices could use unexpected purchases $\left(E_1 - E_0\right)\left(B_1^{10} - B_0^{10}\right)$ to offset shocks to surpluses, and remove predictable components from inflation.

The general case of these formulas is quite complex (Cochrane 2001), suggesting a very interesting job of maturity management for governments that want to stabilize inflation and, with pricing frictions, output.

3 Real rates and sticky prices

The simple models got us quite far, but leave out two crucial questions for monetary policy: its effects on output and real interest rates. By being completely neutral, the simple models allow the Fed to control nominal rates and expected inflation, and to exert a great influence on actual inflation in the presence of long-term debt, but it has no influence on real rates or output. This means that to increase inflation, the Fed raises, not lowers nominal rates. It also means that the conventional view that the Fed operates through real interest rate increases is absent.

Now, again, it is not at all obvious that monetary policy in an interest on reserves regime will have the traditional effects. The mechanism for interest rate increase, and the mechanism for transmission to the price level are utterly different. Still, we need to explore potential real rate and output effects.

3.1 A simple sticky-price model

To that end I maintain a model without *monetary* frictions – reserves still are perfect substitute for overnight Treasury debt, and people hold no money overnight. But I add *pricing* frictions. I add pricing frictions in the simplest possible way. The point here is not to create an empirically successful model of dynamics, but to explore how simple, standard, and transparent theory predicts monetary policy will work.

Here, I use the simplest possible sticky-price model to explore both issues. While one can have many standard objections to sticky-price models, I think it is better to innovate along as few dimensions as possible. So I embed a very standard sticky-price mechanism into a completely cash-free model with fiscal price level determination.

I set the model out in detail in the Appendix. It is a simplification of Galí (1999). Households consume a CES composite good of many varieties. Each household uses labor to produce one variety, and must set its price one period in advance.

The general conclusions are as one would guess. The government debt valuation equation remains, unsurprisingly,

$$u'(c_t)\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j u'(c_{t+j}) s_{t+j}$$

where now P_t must be determined at time $t-1$. You can see right away that since P_t cannot adjust, either $u'(c_t)$ or $E_t u'(c_{t+j})$ must adjust when there is a surplus shock. Real interest rates and output will be affected. The point of the model is to determine which of these possibilities holds, and how P_t is determined.

Since prices are only sticky for one period, marginal utility can only be expected to diverge from the frictionless value for one period. With $\bar{c}$ equal to the frictionless level of consumption and output, we have

$$E_{t-1}\left[u'(c_t)\right] = u'(\bar{c}).$$

This also means that the real interest rate

$$\frac{1}{1+r_t} = E_t\left[\beta \frac{u'(c_{t+1})}{u'(c_t)}\right] = \beta \frac{u'(\bar{c})}{u'(c_t)}$$

is only affected for one period.

Now, I make a second assumption: *Surpluses s_t are also known one period in advance,*

$$s_t = E_{t-1}(s_t).$$

The point of this assumption is to allow me to discount future surpluses at the risk free interest rate, and to avoid a risk premium induced by $cov(s_{t+j}, u'(c_{t+j}))$. I ruled out such risk premia in the endowment economy model by ruling out variation in output, potentially correlated with surpluses. Risk premia in government debt are interesting and add a lot of possibilities to the model. In fact, it is likely that much variation in the value of government debt, like all securities, corresponds to variation in the risk premium rather the riskfree rate, and to understand variations in aggregate demand (the flip side of demand for government debt) over business cycles, we will need to consider variation in the risk premium. But risk premiums also add complexity not needed at this stage.

With this assumption[1], we can write the present-value equilibrium condition as

$$\frac{u'(c_t)}{u'(\bar{c})}\frac{B_{t-1}}{P_t} = \frac{u'(c_t)}{u'(\bar{c})}s_t + E_t\sum_{j=1}^{\infty}\beta^j s_{t+j}. \tag{11}$$

3.2 Analysis

Again, look at expected and unexpected components.

$$\frac{B_{t-1}}{P_t}\left(E_t - E_{t-1}\right)\frac{u'(c_t)}{u'(\bar{c})} = \left(E_t - E_{t-1}\right)\left[\frac{u'(c_t)}{u'(\bar{c})}\right]s_t + \left(E_t - E_{t-1}\right)\sum_{j=1}^{\infty}\beta^j s_{t+j}. \tag{12}$$

Now, P_t is no longer inside the $E_t - E_{t-1}$, so prices cannot adjust to the surplus shock. *All the adjustment to the surplus shock comes by adjustment to $u'(c_t)$ and hence to the real interest rate, since the price level cannot adjust.*

A negative (inflationary) shock to expected future surpluses $(E_t - E_{t-1})\sum_{j=1}^{\infty}\beta^j s_{t+j}$ lowers $u'(c_t)$, i.e. raises c_t, so *the inflationary shock produces a temporary output expansion. The inflationary shock also lowers the real interest rate,*

$$1 + r_t = \frac{1}{q_t} = \frac{1}{\beta}\frac{u'(c_t)}{u'(\bar{c})}.$$

These are the "usual" signs – inflation is preceded by higher output and lower real interest rates.

Writing (12) in the form

$$\frac{B_{t-1}}{P_t} = s_t + \frac{1}{\left(E_t - E_{t-1}\right)\left(1 + r_t\right)}\left(E_t - E_{t-1}\right)\sum_{j=0}^{\infty}\beta^j s_{t+1+j} \tag{13}$$

[1] Algebra:

$$u'(c_t)\frac{B_{t-1}}{P_t} = u'(c_t)s_t + E_t\sum_{j=1}^{\infty}\beta^j E_{t+j-1}\left[u'(c_{t+j})s_{t+j}\right].$$

$$u'(c_t)\frac{B_{t-1}}{p_t} = u'(c_t)s_t + E_t\sum_{j=1}^{\infty}\beta^j s_{t+j}E_{t+j-1}\left(u'(c_{t+j})\right)$$

$$u'(c_t)\frac{B_{t-1}}{p_t} = u'(c_t)s_t + E_t\sum_{j=1}^{\infty}\beta^j u'(\bar{c})s_{t+j}$$

and reminding ourselves of the one-period flow equilibrium condition money in = money out,

$$B_{t-1} = P_t s_t + Q_t B_t, \tag{14}$$

gives insight how a change in the real rate absorbs the surplus shock. With a constant real interest rate, the real value of debt sold in the evening declined when expected future surpluses declined. To match that decline, the real value of debt coming due in the morning declined, as P_t on the right hand side of (13) rose. But that mechanism is now absent, since the real value of debt coming due in the morning B_{t-1}/P_t cannot decline. How can it stay the same in the face of a decline in expected surpluses? The interest rate also declines, so the same expected surpluses have higher value!

Tracking the money in (14) helps to understand this mechanism. We can imagine the government printing up money B_{t-1} to pay off debt at the beginning of period t, money which must be soaked up somehow by the end of time t. With flexible prices, the real value of current and future surpluses was fixed, and at the same nominal price level P_t would not soak up all the dollars after the negative surplus shock. So people tried to buy more goods with their excess dollars. With flexible prices, they just pushed up the prices of those goods until the same real surplus s_t and same real quantity of debt sold soaked up the excess nominal dollars brought in by B_{t-1}.

But now prices cannot rise. People still have more newly-printed money in their pockets B_{t-1} than will be soaked up by surpluses s_t and debt sales. What happens? First, they try to buy more goods and services. With prices fixed one period in advance, this extra "aggregate demand" does in fact lead to greater output. But the greater output does not soak up any money in aggregate. More money spent by the purchaser is received by the seller, and at the end of the day the excess money B_{t-1} relative to its sponges is still there. So, if money holders cannot bid up the price of *goods,* they bid up the price of *bonds* instead. "Asset price inflation," takes the place of nominal inflation. The interest rate decline / bond price rise is just enough so that the excess cash is now all soaked up by bond sales. The real interest rate also limits the rise in output. We can think of the intertemporal first order condition $1/(1+r_t) = E_t\left[\beta u'(c_t)/u'(c_t)\right]$ as connecting the increase in output at time t to the interest rate rise needed to soak up extra cash.

Though now (I hope) obvious in terms of the model, these are unconventional predictions. Without the model, we might have thought that a decline in expected future surpluses, a decline in the government's ability to service its debt, would lead to an *increase* in the interest rate, and a *reduction* in the value of government debt. Instead, interest rates rise and there is no change in the real value of government debt.

Next consider monetary policy. Looking back a period, taking E_{t-1} of (11), with $E_{t-1}\left[u'(c_t)\right] = u'(\bar{c})$ and remembering that p_t is known at time $t-1$, we obtain

$$\frac{B_{t-1}}{P_t} = E_{t-1} \sum_{j=0}^{\infty} \beta^j s_{t+j}. \tag{15}$$

This equation functions much as its flexible price counterpart (6). By varying debt B_{t-1}, the government can control the price level at time t with sticky prices, just as it controlled the expected price level at time t, $E_{t-1}(1/P_t)$, with flexible prices, even with no real fiscal changes $\{s_t\}$. Again, this action is like a share split or currency reform.

Regarding changes in $\{B_t\}$ with fixed $\{s_t\}$ as "monetary policy" and changes in $\{s_t\}$ with fixed $\{B_t\}$ as "fiscal policy," however, we do not change the conclusion that monetary policy can affect the nominal interest rate, but monetary policy cannot affect the real interest rate. The real interest rate r_{t-1} does not enter in to (15), and the choice of B_t does not enter in to the determination in (13) of r_t.

Equation (15) works differently than its frictionless counterpart: P_t is fixed at $t-1$, so the nominal rate and real rate are related by

$$(1+r_{t-1}) = (1+i_{t-1})\frac{P_{t-1}}{P_t}. \tag{16}$$

Thus, "monetary policy" adjustments in B_{t-1} in (15) are still equivalent to a nominal interest rate target, and the government can still target the nominal rate i_{t-1} without affecting fiscal surpluses.

With real rates determined by fiscal shocks at time $t-1$, (and, in a fuller model, real shocks), a fixed nominal rate target will result in price level volatility: If the real rate r_{t-1} rises and the Fed holds the nominal rate i_{t-1} constant, the price level P_t must decline in (16), which will be achieved by fewer bond sales at the fixed nominal rate i_{t-1}. Hence, *a nominal rate target, desiring little price volatility, should, in this model move one for one with the real rate,* which Fed functionaries might interpret as instruction to follow rises and falls in the "natural" rate.

4 A model with output and real interest rate variation

The simple model of the last section assumed that the maturity of government debt and the horizon of price stickiness were exactly equal at one period. Changing that equality – allowing allows the potential for monetary policy to affect real interest rates.

Suppose now that prices must be set *two* periods in advance. This will give the government a period in which to do monetary policy while prices are still stuck. As there are no asset market distortions, the government debt valuation equation remains,

$$u'(c_t)\frac{B_{t-1}}{P_t} = E_t\sum_{j=0}^{\infty}\beta^j u'(c_{t+j})s_{t+j} \tag{17}$$

where now P_t must be determined at time $t-2$. Marginal utility can only be expected to diverge from the frictionless value for two periods.

The general algebra for this case does not yield much intuition, so I present a simple example. Since our interest is in examining what "monetary policy" construed as variation in debt B with no variation in surpluses s can do, I simply fix the surplus path at a constant $\bar{s}$.The flow of near-term surpluses clutters up the equations, so I assume surpluses are zero until period 10, after the intervention is well over. Then we can write the present-value equilibrium condition (17) as

$$\frac{u'(c_t)}{u'(\bar{c})}\frac{B_{t-1}}{P_t} = \frac{\beta^{10-t}}{1-\beta}\bar{s}.$$

Now, if the government sells more B_{t-1} at time $t-1$, P_t cannot change as it did before, because P_t is set at time $t-2$. So $u'(c_t)$ and the real interest rate from time $t-1$ to t, and then from t to $t+1$, must now change in response to monetary policy.

Suppose the government starts on a steady-state path $B_t = \beta^{10-t}\bar{B}$, so $P_t = \bar{P}$, where

$$\frac{\bar{B}}{\bar{P}} = \frac{\bar{s}}{1-\beta}.$$

At time -1, the government suddenly and unexpectedly sells more debt, raising $\bar{B}$ to $(1+\delta)\bar{B}$, and leaving it there, i.e. $B_t = (1+\delta)\beta^{10-t}\bar{B}$. Once prices can change, starting at P_1, prices just go up to $P_t = (1+\delta)\bar{P}$. But we have at time 0,

$$\frac{u'(c_0)}{u'(\bar{c})}\frac{(1+\delta)\bar{B}\beta^{10}}{\bar{P}} = \frac{\beta^{10}\bar{s}}{1-\beta} = \beta^{10}\frac{\bar{B}}{\bar{P}}.$$

$$\frac{u'(c_0)}{u'(\bar{c})} = \frac{1}{1+\delta}.$$

The inflationary rise in B causes a decline in $u'(c_0)$, a little rise in c_0. This will cause a decline and then rise in the real interest rate, since $c_1 = \bar{c} < c_0$, before inflation occurs. *Monetary policy first affects the real rate of interest, then causes an output increase, and then causes inflation.*

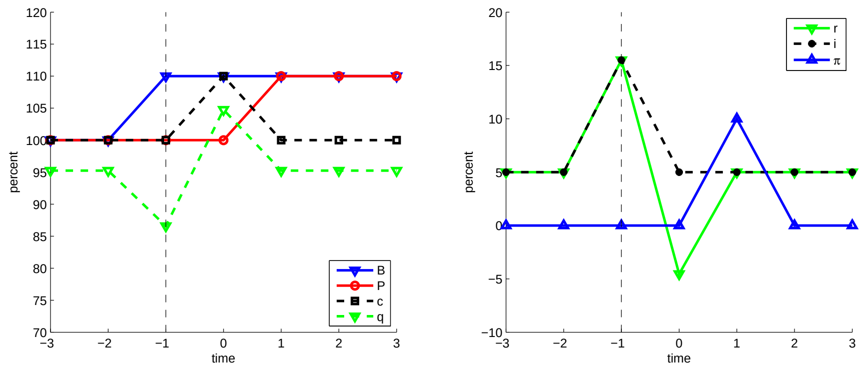


Figure 1: Simulation of an unexpected increase in debt B_{t-1} with no change in surpluses, in a model with prices set two periods in advance.

Figure 1 illustrates. In the left panel, debt B_t takes an unexpected jump from 100 to 110 at time -1. The price level P_t is stuck for two periods, so cannot change until time 1, when the inflation (price level jump) actually happens. Consumption – I use log utility so $c_t = 1/u'(c_t)$ – does not move at time -1, but makes a one-time jump at time t. The interest rate r_t, shown in the right panel, and the real bond price $q_t = 1/(1+r_t)$ shown in the left panel, track consumption growth. At time -1, consumption is expected to grow, so the interest rate rises and the bond price declines. At time 0, consumption is expected to decline, and the bond price rises and the real interest rate fall. Inflation is positive for the period of the one-time jump in prices. The nominal interest rate, by contrast, only blips up for one period while the real interest rate is higher. The lower real rate at time 0 is matched by higher expected inflation, producing no change in the nominal rate. So this model is still "Fisherian," albeit with a one period lag between the nominal rate and inflation.

One can also read Figure 1 as a simulation of what would happen if the Federal Reserve were to raise its nominal interest rate target unexpectedly for one period at time -1, and then reverse it, following the path outlined in the right hand panel, and were to auction bonds in order to achieve its result, with no fiscal cooperation. Prices being stuck for two periods, in period -1, this action must and does just raise the real rate of interest. Raising the real rate of interest in this economy, however, does not depress consumption by lowering c_{-1} so consumption may grow to time 0. In this economy, raising the real rate of interest leaves c_{-1} alone and raises c_0.

In sum, we see that in a model with price frictions, but no monetary frictions at all, and relying on fiscal backing to determine the price level, "monetary policy" construed as variation in the quantity of debt or variation in a nominal interest rate target, with no effect on surpluses whatsoever, can induce real interest rate and output dynamics. The nature of the real interest rate effect, and the nature and timing of output effects, depends sensitively on the timing of the sticky-price mechanism. I do not mean the above exercise to be at all "realistic," or even to represent correctly the sign of monetary policy effects. In addition, anytime one looks at data, one must recognize that historical events mix monetary and fiscal shocks. Specifying realistic non-neutralities and a realistic specification of what monetary and fiscal policies do to explain data in this sort of model remains an open question.

5 Comparison with a new-Keynesian model, the importance of fiscal anchoring.

A natural reaction at this point is, wait a minute. We have a whole range of models which specify the reaction of the economy to monetary policy, based only on nominal interest rate control by the Federal Reserve, with no mention of quantity limits or fiscal backing: The whole New-Keynesian Taylor-type rule DSGE literature. Why not just reference those and go on to other questions.

In fact, however, this class of models *does* rely heavily on fiscal backing. When you look at them, these models in fact generate inflation predictions by imagining that monetary policy leads to some strong fiscal policy responses. The results of those models depend crucially on this fiscal backing.

The point is *not* that one should evaluate monetary policy holding fiscal policy $\{s_t\}$ constant. The point is, that one should evaluate monetary and fiscal policy together.

5.1 A very simple model

To make these points, consider the absolutely simplest new-Keynesian model, as presented in Woodford (2005): A Fisher equation (first order condition for intertemporal allocation of consumption, in an endowment economy, as above); a Taylor-type rule by which the Fed sets the nominal rate, and a serially correlated monetary policy shock, with which we can evaluate the effects of monetary policy,

$$i_t = r + E_t \pi_{t+1} \tag{18}$$

$$\begin{aligned} i_t &= r + \phi_\pi \pi_t + x_t \\ x_t &= \rho x_{t-1} + \varepsilon_t. \end{aligned} \tag{19}$$

The equilibrium condition is

$$E_t \pi_{t+1} = \phi_\pi \pi_t + x_t$$

There are multiple equilibria. Any

$$\pi_{t+1} = \phi_\pi \pi_t + x_t + \delta_{t+1};\ E_t(\delta_{t+1}) = 0 \tag{20}$$

is a valid solution.

The New-Keynesian tradition sets $\phi_\pi > 1$. All but one solution now explodes, $\|E_{t+1}\pi_{t+j}\| \to \infty$. Ruling out nominal explosions, one selects the forward-looking solution

$$\pi_t = -\frac{1}{\phi_\pi - \rho} x_t$$

and interest rates thus follow:

$$i_t = -\frac{\rho}{\phi_\pi - \rho} x_t$$

Equivalently, this equilibrium chooses the shock

$$\delta_t = -\frac{\varepsilon_t}{\phi - \rho}. \tag{21}$$

The new-Keynesian model in this sense affects inflation by forcing the economy to jump to a different equilibrium.

Figure 2 presents in the blue lines the response to a monetary tightening $\varepsilon_1 = 1$ in this simple canonical model. The monetary policy shock x_t is positive and slowly declines following the AR(1) pattern. Inflation jumps down; the tightening lowers inflation as we imagine it should. The actual interest rate also falls, which seems like a strange sort of tightening. But the actual interest rate falls *less* than inflation. This represents "tightening" relative to the Taylor rule with $\phi_\pi > 1$.

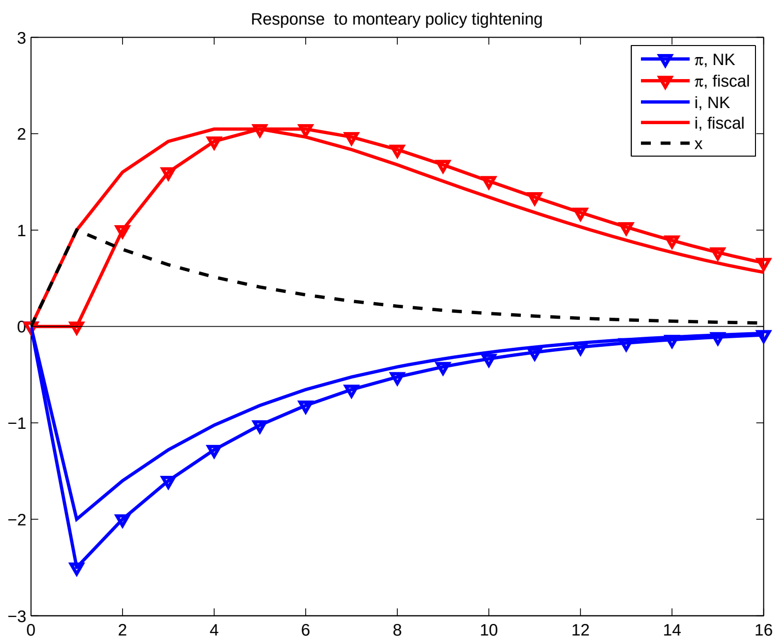


Figure 2: Responses to a monetary tighteing in the standard and fiscally-constrained solutions of a new-Keynesian model. $\rho = 0.8$, $\phi_\pi = 1.2$ for the new-Keynesian model and $\phi_\pi = 0.8$ for the fiscal solution.

What happens to the valuation equation for government debt in this model

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}? \tag{22}$$

It's there; it just got brushed in to the footnotes with an assumption that the Treasury will always pass lump sum taxes $\{s_t\}$ to validate whatever solution we chose. The inflation drop at time $t = 1$ is an *unexpected* drop, as (21) makes clear. As we have seen, the only way to produce an unexpected drop in inflation by (22) is to imagine a change in fiscal policy,

$$\frac{B_{t-1}}{P_{t-1}} (E_t - E_{t-1}) \left(\frac{P_{t-1}}{P_t} \right) = (E_t - E_{t-1}) \sum_{j=0}^{\infty} \beta^j s_{t+j}. \tag{23}$$

Thus, to produce the unexpected -2.5% inflation in response to "monetary" policy shown in Figure 2, we *must* also think that fiscal policy produces a 2.5% increase in the net present value of primary surpluses, to validate a 2.5% increase in the real value of government debt, and that people know this and expect it. In the US context, with \$12 billion dollars of outstanding debt, that means that the Treasury will come up with about \$300 billion of extra tax increases or spending cuts, in present value terms, to validate the monetary policy. Larger stabilizations would need more fiscal backing.

From the point of view of (22), and accepting the private sector budget constraint that "aggregate demand" must equal a change in demand for government debt, in fact, the *mechanism* by which "monetary policy" produces the downward jump in inflation is by inducing this fiscal reaction.

However, the path of inflation after time $t = 1$ requires no fiscal coordination. As we have seen, the government can produce any *expected* inflation by variations in debt $\{B_t\}$ with no changes in surpluses.

One may ask, what if that fiscal backing is not forthcoming? Or, what if people stop *expecting* it? Equations (23) and (20) allow a nice view of this conundrum: *we can index all the multiple solutions to the new-Keynesian model by the implied fiscal backing.* For example, the case of no fiscal response, $(E_t - E_{t-1})\sum_{j=0}^{\infty}\beta^j s_{t+j} = 0$, that I studied above, is the case $\delta_{t+1} = 0$.

Figure 2 also includes this "fiscal-neutral" solution to the model. Now, if $\phi_\pi > 1$ this solution would produce explosive dynamics. In that equilibrium choice, the Fed should follow $\phi_\pi < 1$. I also plot a solution for $\phi_\pi = 0.8$ rather than $\phi_\pi = 1.2$ One might object that regressions show $\phi_\pi > 1$, but in fact in this simple model the regression of i_t on π_t yields $\phi_\pi = 0.8$ in both New-Keynesian and fiscal solutions of the model, so they are observationally equivalent. And on first principles, using $\phi_\pi < 1$ is actually more intuitive. Our Fed makes lots of noises about how they like to stabilize the economy and inflation, they do not destabilize the economy and threaten hyperinflation to produce determinacy.

In the fiscal-neutral solution of Figure 2, plotted in red, interest rates do rise. Inflation does not jump in the period of the shock – that's how we identified the equilibrium choice. This actually sounds pretty reasonable given the data, in which inflation seems pretty sluggish, rather than seeing price level jumps on the same day as FOMC announcements. Then interest rates follow obvious dynamics generated from the shock and inflation.

Now, the fiscal solution gives positive inflation in response to monetary tightening. Isn't this bad –aren't we supposed to see lower inflation in response to monetary tightening, as the new-Keynesian solution showed (granted that measuring "tightening" might be hard given data from that model)? No. This is a purely frictionless model. Real rates are constant, and there is no mechanism for real rates to lower "demand." In a totally frictionless model, all the Fed can do when it raises the nominal rate is to raise expected inflation. So of course raising the nominal rate raises inflation. They mystery here is, how did the new-Keynesian solution produce a downward jump in inflation from a completely frictionless model, with fixed real rate, output, and super-neutrality, yet somehow raising the nominal rate lowers inflation, instantly?

The bottom line is not realism. The important point is that solution choice, and the nature of fiscal backing is vitally important to the model's predictions. Whatever the right answer, the nature of fiscal-monetary coordination is central to the model predictions.

The answer really has nothing to do with monetary policy. The new-Keynesian prediction fundamentally generates the reduction in inflation, from a perfectly frictionless model, by a model of fiscal - monetary coordination, a complex Sargent-Wallace game of chicken, in which the Fed by threatening hyperinflation is able to convince the Treasury to produce a sharp fiscal contraction, which produces the disinflation.

This is not necessarily a criticism. My deeper point is that in our brave new world without monetary frictions, we ultimately have to anchor the value of money in its fiscal backing; and since historical events seem to combine "monetary" – changes in debt – and "fiscal" – changes in surpluses – we only make progress by examining both. But a deep look at the monetary -fiscal coordination underlying its predictions is far from the standard mode of analysis in new-Keynesian models.

5.2 The three equation model

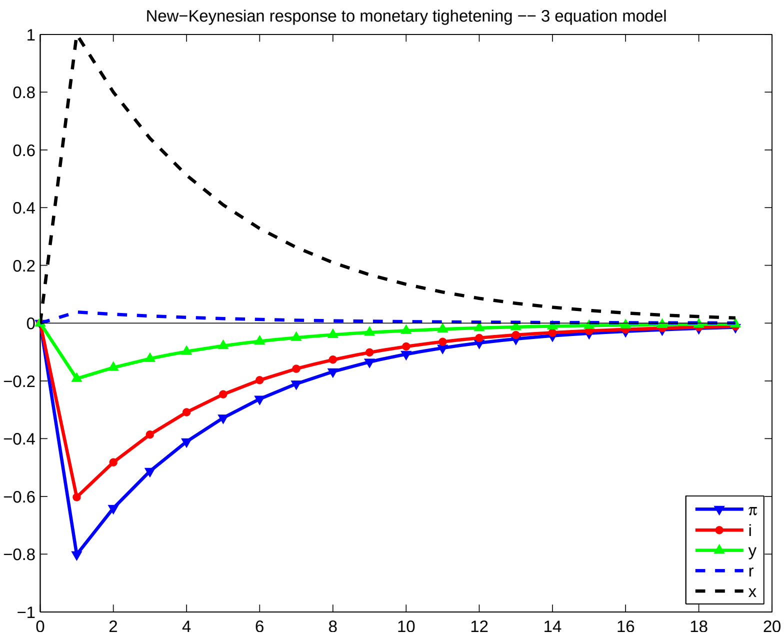
The system (18)-(19) may seem too simple to examine this issue. But the same point holds – all new-Keynesian models pick one of multiple solutions by engineering a nominal explosion; all of those solution choices can also be indexed by the implied fiscal commitment; jumps in inflation imply jumps in fiscal policy; and model predictions are very sensitive to the assumed fiscal backing.

To demonstrate this point, I examine solutions to the standard three-equation new-Keynesian model,

$$\begin{aligned} y_t &= E_t y_{t+1} - \sigma \left(i_t - E_t \pi_{t+1} \right) + x_{dt} \\ \pi_t &= \beta E_t \pi_{t+1} + \gamma y_t + x_{\pi t} \\ i_t &= \phi_\pi \pi_t + x_{it} \end{aligned}$$

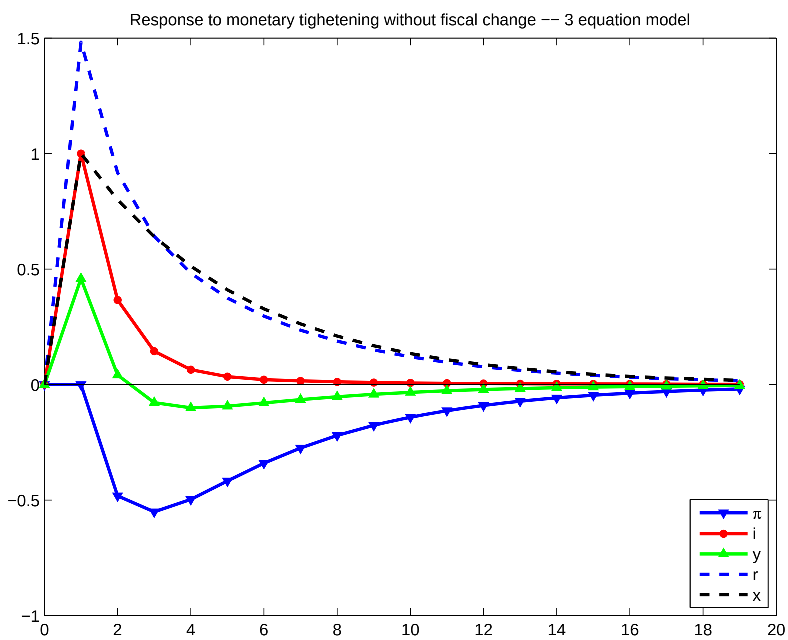
$$\begin{aligned} x_{dt} &= \rho_d x_{dt-1} + \varepsilon_{dt} \\ x_{\pi t} &= \rho_\pi x_{\pi t-1} + \varepsilon_{\pi t} \\ x_{it} &= \rho_i x_{it-1} + \varepsilon_{it} \end{aligned}$$

Figure 5.2 presents the results. As one might expect, and similarly to the simple model of Figure 2, the monetary tightening lowers inflation and output. Again, however, the solution depends on a jump downwards in inflation, which requires a fiscal tightening.



Response of standard new-Keynesian model to a monetary policy shock.

To compare, I present a "fiscal-neutral" solution. Here again, I just picked the equilibrium of the same model in which $(E_t - E_{t-1})\,\pi_t = 0$. I also changed ϕ_π to a value below one, so the responses would be stable. Figure 5.2 presents the results.



Fiscal-neutral solution to the three equation model.

This change produces radically different responses. Inflation cannot now "jump" down during the period of the shock. The tightening now produces an actual rise in nominal rates. This is a rise in real rates as well, with expected inflation declining. Output has two influences: the higher real rates want less output today than tomorrow, but higher inflation today than in the future wants a boom today relative to tomorrow. The Phillips curve wins (and you can see how some lags in inflation might help here). I'd judge the inflation response as sensible, the output response as not – we'd expect a general decline in output to match what people think these responses should look like.

The point is not that this issue is settled. The is that the responses are *very* different across two solutions of the same model, indexed by different views of monetary-fiscal policy coordination.

This issue is one that one can settle empirically. In this model, with one-period debt, the contemporaneous response of inflation to any shock measures the fiscal response – the response of $E_t \sum_{j=1}^{\infty} \beta^j s_{t+j}$ – to that shock. That measurement is, of course, conditional on the model. In models in which inflation is determined one or more periods in advance, inflation itself cannot move – $(E_t - E_{t-1})\,\pi_t = 0$. But other state variables do jump, and the corresponding jump in fiscal response should be measurable as well.

A disclaimer: properly integrating fiscal backing into models of this sort is more complex than simply adding the frictionless valuation equation, as I have implicitly done here to make a clear illustrative calculation. At a minimum, interest rate changes (and later, risk premium changes) must be reflected in the value of government debt. The correct measure is $\sum_{j=0}^{\infty} m_{t,t+j} s_{t+j}$, and real interest rates can and do change the value of government debt without any change (or even with contrary changes) in expected surpluses. The details of asset purchases and budget constraints, and the nature of price stickiness, need to be specified explicitly, as I did in the Appendix for the model with prices stuck two periods in advance, along with the maturity structure of government debt and state-contingent changes in that maturity structure.

6 Can the Fed control the balance sheet and interest rates?

The sort of "monetary policy" I have described so far is a great deal more ambitious than what the Federal reserve or other banks contemplates. Examining (9), reproduced here,

$$\frac{B_{t-1}}{P_{t-1}} \frac{1}{1+i_{t-1}} = E_{t-1} \sum_{j=0}^{\infty} \beta^{j+1} s_{t+j},$$

fixing the interest rate i_{t-1} essentially replaces government debt auctions with a fixed price. A central bank in charge of such a policy must pay i_{t-1} on reserves, or charge i_{t-1} on any lending. But then the quantity of short term debt B_{t-1} must be endogenous. The changes may not be enormous – to induce 1% extra inflation, the government must sell only 1% extra debt (or commit somehow to 1% less present value of surpluses). But the quantity must be endogenous.

In this equation, the quantity B_{t-1} makes no distinction between reserves at the central bank and Treasury debt of the same maturity. If we interpret open market operations or quantitative easing operations as changes in the form (reserves vs. treasury debt) of government debt of the same maturity, with no change in fiscal expectations, those operations have no effect at all. One needs additional liquidity frictions, segmentation, etc. to achieve any effect. One might interpret purchases as a change in maturity structure of government debt, by the mechanism of section (2.4). But the issue going forward is whether the central bank can affect interest rates and inflation by conventional open market purchases trading short term treasury debt for reserves with not much effect on overall maturity.

In other institutional arrangements, central banks might be able directly to issue Treasury debt without fiscal repercussions, the equivalent of helicopter drops. But not ours. Part of the limitations on Federal Reserve authority in exchange for its independence is that it must be a bank, and always buy something in return for reserves. When the Fed buys mortgage-backed securities, it would seem that government debt B_{t-1} rises, but the mortgage backed security is, in principle, an asset whose payoff contributes to fiscal surpluses $\{s_{t+j}\}$ in the consolidated government balance sheet, i.e. including reserves as government debt in B_{t-1}. Even in unsecured lending to banks or other financial institutions, the loan appears as an asset and if made properly has no effect. The one thing our central bank is completely forbidden to do is the one thing that most economists think would most clearly affect inflation – a helicopter drop of money or short-term debt. That is fiscal policy, writing checks to voters, which only the Treasury may do. (If you're not convinced that helicopters are fiscal policy, consider the converse: a vacuum cleaning, i.e. a lump-sum removal of money.) So to the extent that the Federal Reserve can increase B_{t-1}, in the context of this simple accounting, it also increases the present value of surpluses by exactly the same amount with no effect on the price level.

Now, if the central bank simply announces a higher interest rate on reserves *and* an unlimited quantity, "we pay 5% on reserves, come and get it. Give us your treasuries, we will create reserves and pay 5% interest," then clearly the central bank can control the nominal interest rate. The interest rate on treasuries must rise to 5% by arbitrage. But then the central bank potentially loses control of the balance sheet.

But our Federal Reserve does not plan to lose any balance sheet control. It plans to pay interest on reserves *and* to control the size of the balance sheet, by determining the quantity of bonds it will buy and sell at market prices. The question is whether, in this arrangement, a higher interest rate on reserves will affect treasury and other rates.

If you decide that gardeners are underpaid, and decide to pay your gardener \$50 per hour, your gardener will be pleased, but this action will not raise the wages of all gardeners to \$50 per hour. You have to offer to hire any number of gardeners at \$50 per hour, and be prepared to hire a lot of gardeners. Even if Wal-Mart decides to pay all its associates \$50 per hour, if it also fixes employment, then it is not clear that the higher wages will spill over from the lucky few who have jobs to the rest.

However, the Fed is larger, and capital markets are capable of a more ruthless level of competition, so it is possible that interest on reserves can raise all interest rates even with a quantity restriction.

Figure 3 illustrates the flow through the banking system. Now, suppose the Fed simply raises the interest rate on reserves with no open market purchases at all. Banks and depositors are competitive. Though banks cannot collectively increase their holdings of reserves, they can try to attract other depositors. If a depositor in bank A switches to bank B, then bank A delivers reserves to bank B, and earns the difference between reserves and deposits. Competition for depositors, then, should drive the deposit rate up to the rate on reserves, minus costs, even if the quantity of reserves is fixed.

Similarly, though banks and individuals cannot collectively exchange treasuries for reserves they can each do so individually. If the rate on reserves exceeds that on treasuries, bank A will offer to sell Treasuries to bank B in exchange for reserves. Bank B will obviously not sell at anything other than the rate it can get on reserves. Banks will try to sell Treasuries to private parties; those parties will deliver bank deposits in exchange for the Treasuries, and the bank holding the deposits must deliver reserves. Also, individuals will all try to sell treasuries in return for interest-paying bank deposits. In this way, the interest rate on treasuries and other loans should move toward the interest rate on reserves. However, the banking system is less than perfectly competitive. Banks

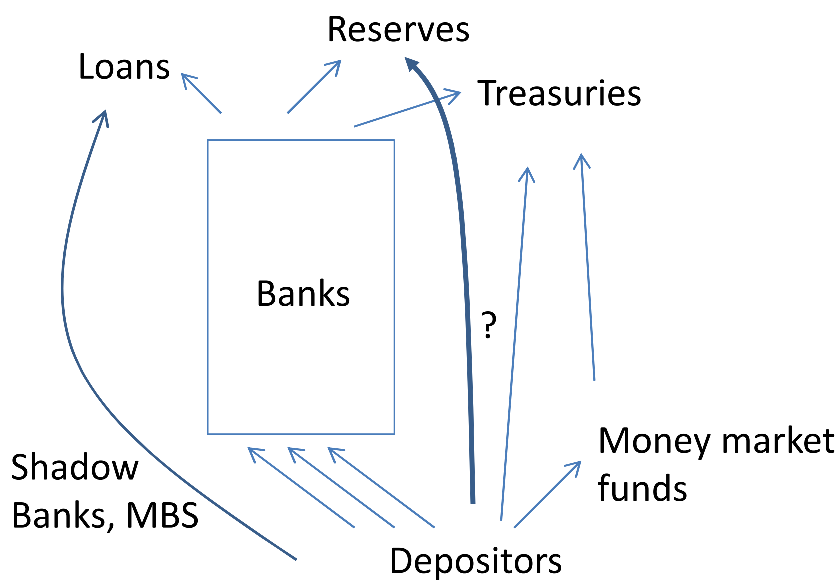


Figure 3: Monetary system diagram, to answer whether raising interest on reserves will raise treasury and mortgage rates without balance sheet expansion.

revel in the "stability" of their deposit base. Reserves already are paying more than Treasuries. So the arbitrage relationships may not work as seamlessly as I have supposed, and the central bank ought to be prepared to give a party where nobody shows up, that raising interest on reserves *without* opening up the balance sheet has less effect to other interest rates than supposed.

Why not Walmart or your gardener? If labor markets were perfectly competitive as I have outlined for banks, then an associate earning \$50 per hour would subcontract his or her job for \$45 per hour, as a bank takes deposits and holds reserves, and the competitive market in subcontracting would lead to economywide wages of \$50 per hour. Obviously, one has to be big to make this work, and markets have to be competitive. Reserves are only \$2.6 trillion relative to \$10 trillion or so of short-term debt, \$18 Trillion of government debt, and larger amounts of corporate debt and mortgage-backed securities, to say nothing of international debt, so whether the Fed is large enough and markets competitive enough is a bit of an open question.

We still need the overall quantity of debt to rise however. How does the Fed paying greater interest on a segment of B_{t-1}, or even buying one identical segment of B_{t-1} in return for another identical segment of B_{t-1} end up raising the overall level?

This will happen if, by arbitrage, the interest rate rises. If the interest rate rises by 5%, the real bond price falls by 5%, and at the next Treasury auction, if the Treasury wants to raise the same *real* amount of resources $E_{t-1}\sum_{j=1}^{\infty}\beta^{j+1}s_{t+j}$ by selling debt at the end of time $t-1$, it will have to announce a larger quantity B_{t-1}.

One might have worried that the Fed would have to completely surrender the balance sheet, and that it might buy all the outstanding debt and need to pressure the Treasury to issue more. This analysis suggests this is not a necessary worry. While opening up the balance sheet will undoubtedly help arbitrage relations between reserves and other debts to happen more quickly, the Fed can, if it wishes, maintain some control of the balance sheet.

Again, there is no particular reason to do so. Once short term treasury debt and reserves are perfect substitutes, the size of a balance sheet long short-term treasury debt and short reserves is irrelevant.

This analysis also suggests a classic sign that may flip in the interest on reserves regime. When the Federal Reserve wants to tighten, to raise interest rates, it is most likely that this action will be accompanied by an *expansion* of the balance sheet. We are used to the opposite sign. We are used to an entirely different mechanism.

I have stated the question, whether the Fed can affect interest on other assets by changing the interest on reserves, i.e. whether arbitrage mechanisms are strong enough so that the presumed desired outcome of all interest rates rising together can happen without losing balance sheet control. The opposite phrasing is just as interesting. The standard view of the Fed's operating procedures in the IOR regime considers interest on reserves different from the Treasury rate and the size of the balance sheet as separately controllable items. The same considerations address whether this is even possible. *Can* the Fed pay more or less on reserves than the going rate for Treasuries, while limiting the size of the balance sheet? Or will arbitrage relationships force the Fed to pay interest equal to treasuries for any balance sheet in the liquidity-satiation range? In the traditional mechanism we did not regard the size of reserves and the interest spread between reserves and Treasuries as separate policy levers; the Fed manipulated either one to control the other on a stable curve. The arbitrage assertion simply says that this constraint also applies in the satiated-liquidity region where the interest spread must be zero.

In sum, the question is just how many policy levers the Federal Reserve has among the level of interest rates, the spread between Treasury rates and interest on reserves, and the size of the balance sheet. If the above arbitrage relations work, any balance sheet in the satiated-liquidity range is consistent with zero spread, but there can be no spread. Then, the Fed can control the level of nominal rates and the size of the balance sheet – though the latter is irrelevant. The Fed will undoubtedly be tempted to adduce various frictions and use more instruments.

6.1 The reverse repo program

The arbitrage relations between treasuries and reserves outlined above would obviously hold better if agents other than banks could participate in the reserves market. Such an arrangement would also be of large benefit to my larger goal of financial stability. Money market funds should be able to invest in reserves. We should at least *allow* narrow banking!

The fact that the interest rate on reserves (0.25%) is, as I write, larger than the interest rate on short-term treasuries (0-0.1%) suggests that intermediation through banks is less than perfect. It also suggests that currently reserves are *less* liquid than Treasuries. Anyone can buy and sell Treasuries, but only banks can hold reserves. Even if you think that open market operations are effective by changing the liquidity composition of government liabilities, it is possible that banks are so satiated in reserves, and sufficiently uncompetitive in the post Dodd-Frank era, that an open market purchase, increasing the size of the balance sheet, *reduces* "liquidity" and is therefore contractionary. The common substitution of the phrase "the Fed increased [or 'injected'] liquidity" for "the Fed increased reserves, buying treasuries" is simply wrong, and may even have the sign wrong.

The Federal Reserve has announced in interesting step in this direction, the reverse repo program. Under this program, qualified non-bank financial institutions would be allowed to engage in repo transactions with the Fed.

The counterparty may lend to the Fed. To do this, the counterparty writes a check to the fed, drawn on the counterparty's bank account. The bank must then deliver reserves to the Fed. The Fed creates a new account in the name of the counterparty, who now holds reserves. Since the counterparty is lending to the Fed, the Fed posts collateral in the form of its securities.

The net effect of this operation is to transfer reserves held by the banking system, and backing bank deposits, into reserves held directly by non-bank financial institutions. The overall quantity of reserves and the size of the balance sheet do not change.

Reading it this way, it becomes a useful tool for encouraging arbitrage between reserves and bank accounts by a class of outside investors. It does not do anything to make the overall size of the balance sheet responsive. For example, this is quite distinct from a policy of endogenous open-market operations: the Fed offers 5% and anyone can sell Treasuries to the Fed and receive reserves that pay 5%. I think the Fed is convinced for some reason that it really wants to control the size of the balance sheet, but encourage the kind of competition for a limited supply of reserves that I described above.

6.2 Reserves for all

Given the analysis so far, that the size of the balance sheet is irrelevant, when consisting of short-maturity treasuries, and that reserves which pay market interest are a very worthy asset, one can conclude that the Fed should go much further than the reverse repo program. Why not let any business or individual have reserves? Why structure lending to the Fed as a repurchase agreement? Who in the world wants collateral when lending to the Federal Reserve, which practically by definition cannot go bankrupt, at least without a default on the same treasuries that constitute the collateral? Why worry about the size of the balance sheet and structure the program so that overall reserves are unchanged?

The main obstacles are not economic, I think, but political and the habits of old institutions. The Federal Reserve is used to being a bank for banks, and not dealing with the public. I suspect that like any government agency, it is loath to undercut private competitors by providing an excellent substitute for very profitable bank deposits. It kept interest on reserves at 0.25%, not zero, with the explicit aim of keeping money market funds in business. I suspect the Fed offers collateral that nobody needs because the existing habits and market mechanisms for private short term lending include collateral in the form of repurchase agreements. The Fed would also require collateral for the expansionary side, lending to private non-banks. There is a quite reasonable political case against a large balance sheet, described below.

There is a good case that the Fed does not want to bother with retail transactions, and does not have a comparative advantage in servicing retail customers. That consideration however still suggests a much wider set of intermediaries could have access to reserves.

More fundamentally, the Treasury can help. What the Fed, for institutional reasons, cannot offer, the Treasury can. *The Treasury should offer fixed-value, interest-paying, electronically transferrable, arbitrarily divisible treasury debt, with the interest rate either set at auction or (better) set equal to the interest the Fed is paying on reserves.* That innovation offers the same economic benefits of a large Fed balance sheet, without the institutional disadvantages of the balance sheet itself.

7 Fiscal and monetary policy in recent history

Fiscal-monetary coordination is not something totally new that will gain force only when reserves pay interest with a large balance sheet. In fact, fiscal-monetary coordination is a larger piece of recent monetary history than is currently appreciated. Or new world is really just a continuation of the world that has already emerged. But, it poses some new and unanticipated limitations on monetary policy.

7.1 Sargent and Wallace reconsidered

In focusing our attention on the fiscal backing of monetary policy, we have to confront Sargent and Wallace (1981) and the conventional interpretation of the 1980s disinflation. Sargent and Wallace made perhaps the most famous wrong forecast in the history of monetary economics. At the time of the Reagan-Volcker disinflation, Sargent and Wallace pointed out that disinflation with high deficits could not work. Using a framework similar to what I have used above, they claimed that the large deficits would inevitability lead to seigniorage and a return to inflation.

Their analytical framework is perhaps one of the most famous innovations in monetary economics of the last 40 years, inaugurating the renewed attention to fiscal-monetary coordination. But their forecast was wrong: The disinflation stuck, without big tax increases or spending cuts they advocated.

The episode is important for a second reason. The inflation and disinflation of the 1970s and 1980s is the central experience, conventionally interpreted to say that monetary policy alone gone wrong creates inflation, and done right can cure inflation. The mainstream treatments of the episode, from Clarida, Gali and Gertler's (2000) shift in Taylor rule regression coefficients, Taylor's own (1999) analysis, and Sargent's (2001) "Conquest of American inflation" emphasize a purely monetary story without much fiscal cooperation.

Now, I have sold a fiscal analysis on the change to interest-paying money and interest on reserves. But there were already so many liquid assets, the move to interest-paying private money market funds, short term commercial paper and repurchase agreements was well underway, the Fed was already targeting interest rates, and the quantity of reserves was already tiny. One can make a good case we were already living in an essentially frictionless regime, and we should regard the change to the kind of analysis I have done for the perfectly frictionless regime as describing the broad features of the prior era, with perhaps some small liquidity effects tacked on. So, examining fiscal-monetary coordination in this episode is doubly important.

Figures 4, 5 and 6 lay out some important features of the episode. Figure 4 shows interest rates and inflation. Prior to 1980, the 3 month treasury bill rate was almost always below the inflation rate. For a brief period in 1980-1981, the 3 month rate rose above inflation. Inflation promptly fell like a stone in 1982. But a period of high ex-post real interest rates followed throughout the 1980s, interrupted only by the quasi-recession of 1987, and the three subsequent actual recessions. Long-term rates indicated an expectation of return to high real rates, or a large risk premium. In any case, they moved little over the recessions. I return to the long downward trend below. The point here: The conquest of inflation entailed a long period of high real interest rates.

Next, Figure 5 presents some of the fiscal history of the period. I measure deficits as the amount the Federal government actually borrows in credit markets. The picture makes one point very clear: *most of the "Reagan deficits" were due to higher interest costs on the debt rather than unusually*

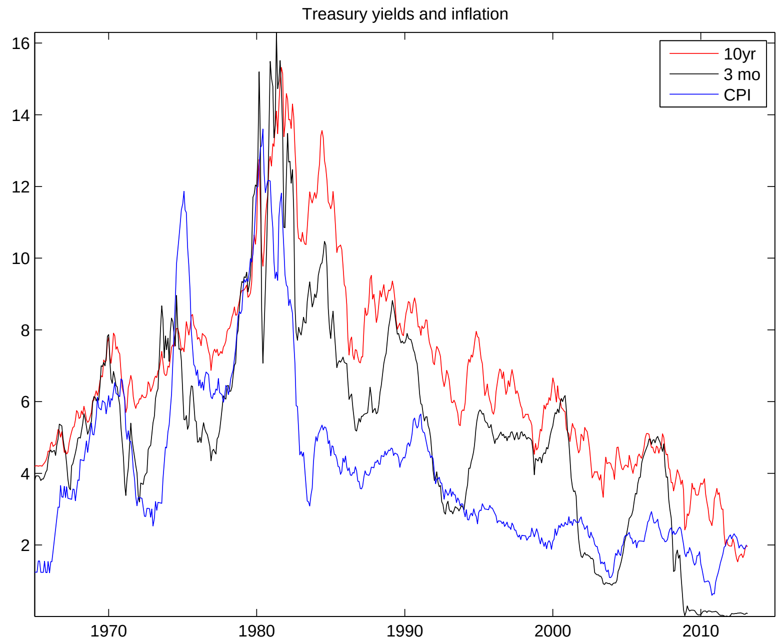


Figure 4: Treasury yields and inflation. Inflation is change from a year ago in log CPI.

large primary deficits. Figure 6 makes the same point in another way, plotting real primary deficits (net of interest costs) along with detrended GDP. The Reagan era primary deficits were not that large. Moreover, Figure 6 emphasizes that the largest driver of primary deficits is the state of the business cycle. As GDP falls tax receipts fall and some spending kicks in automatically. The Reagan deficits were not that large compared to the size of the recession, and thus not likely to spur a change in evaluation of long-run fiscal policy.

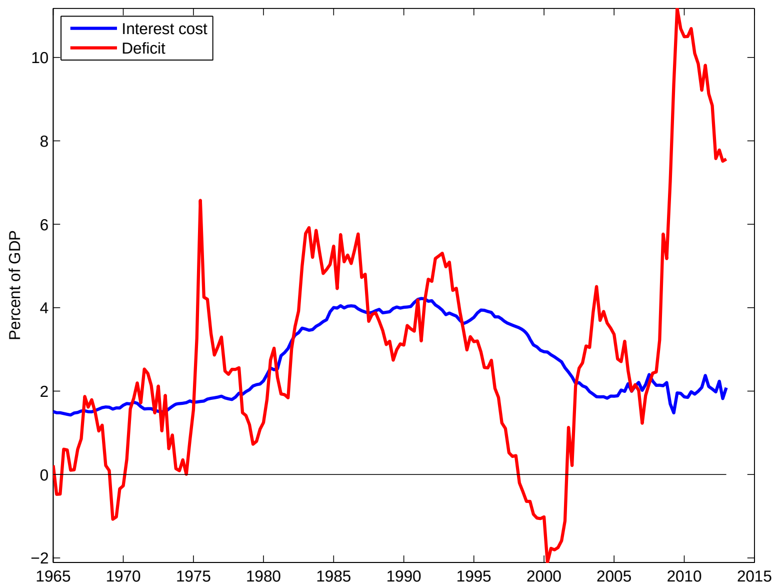
Figures 4 and 5 are revealing of the fiscal costs of this disinflation. As Figure 4 shows, investors who bought long-term bonds at high interest rates in the late 1970s, expecting large inflation, got a windfall as their bonds were paid off with much lower inflation. The P_t in B_{t-1}/P_t declined unexpectedly. Someone had to pay for this windfall in higher subsequent surpluses.

Figure 5 reveals a second, larger, fiscal cost of disinflation. In the conventional narrative of a stabilization, the high real interest rates were the crucial monetary policy intervention needed to bring down inflation. But much US debt is relatively short term, so must be rolled over every few years. As the US rolled over a (then) fairly large 20% of GDP stock of debt, the real interest costs of financing the deficit rose. Figure 5 suggests that these interest costs were about 2% of GDP for a decade, so a cumulative 20% of GDP. Those costs came from somewhere as well. Interest costs result in more debt being sold, and that debt too must be paid with higher future surpluses.

So, where did the future surpluses come from, the ones that Sargent and Wallace warned would lead to seignorage. Figure 6 gives the answer. Starting in the late 1980s, and suggestively coincident with the tax reforms, the US economy started to grow much faster than before. That growth have led to much higher surpluses, which eventually paid down not only the debt that would already have been outstanding, but also the extra debt incurred to pay off a windfall to inflation-era bondholders, and a decade's higher interest costs.

That the debt was paid off does not mean it was costless. That 20% of GDP are real resources,

Figure 5: Interest costs and Federal Deficit. BEA table 3.2 federal govenerment current receipts and expenditures. Interest payments and net lending or net borrowing. Both series as a percent of GDP.



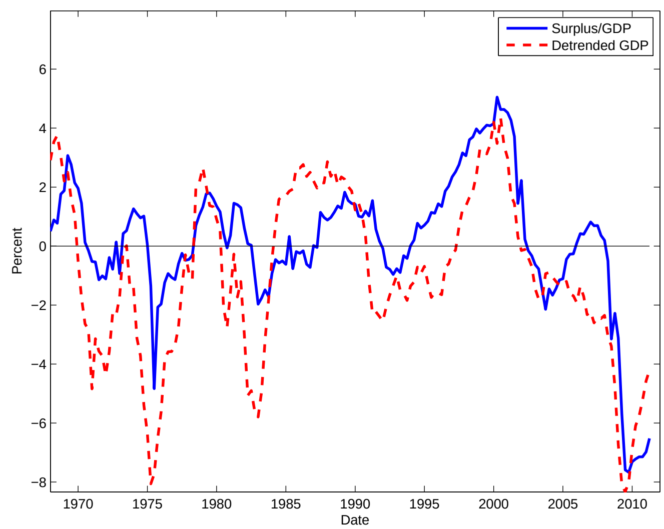


Figure 6: Real primary surplus / real GDP, and detrended real GDP. GDP is detrended with a log-linear trend fit through the previous 15 years.

which could have been put to other uses.

So, the answer to the Sargent and Wallace question is this: They had the right equation. But in making a forecast, they did not foresee that a spurt in real growth would provide the necessary surpluses to finance the Reagan-Volcker disinflation.

The larger lesson is important. The 1980 disinflation was a *joint monetary-fiscal* stabilization. Ending inflation cost something on the order of magnitude of 20% of GDP. The US was able in the short-run to borrow that money, and bondholders somehow had faith they would be paid back and it would not be inflated away. The bond buyers were right. It was paid back, and they enjoyed an unprecedented run.

But it did not have to happen that way. Absent the fiscal coordination; absent the large increase in $\sum \beta^j s_{t+j}$ that in fact occurred, the 1980 stabilization could have and would have failed. Just as so many other purely monetary stabilizations without fiscal reforms have failed.

7.2 Fiscal limits to current monetary policy

This stylized history leads to a central concern for our interest-on-reserves regime. Suppose our Fed decides to tighten, and wishes to raise interest rates to 5%. Suppose further, that as in the 1980s, this is a rise in real interest rates; that inflation does not just jump to 5%, that the contraction is as intended, effective in limiting or decreasing inflation. In such a scenario, the 2% of GDP interest costs we see in the 1980s are nothing like what we will see now. For the debt-to-GDP ratio was only 20% in 1980. Now it is 100%, and that's just official Federal debt, not including credit guarantees. If the US government refinances \$18 trillion of debt at 5% real interest rates, that means \$900 billion of deficit annually. Those additional interest costs must either be paid now, with \$900 billion per year of current taxes or less spending – more than the current deficit – or by accumulating debt that much faster and somehow paying it off in present value terms as we did in the 1980s.

The nice footnotes about Treasury being sent a bill for lump sum taxes to validate a money-dominant regime are likely to be put to a severe test with this experiment. I think it more likely that Congress simply refuses, and takes away the Fed's independent authority to set interest rates, and thereby to impose interest costs on the budget.

This outcome happened before, the last time the US had a debt of this magnitude, at the end of World War II. Congress simply told the Fed to target long term rates at 2.5%, in order to lower financing costs. Congress could have the same response to a Fed-imposed \$900 bill for interest costs. This fiscal-monetary interaction, not mark to market losses on the Fed balance sheet, or reduction Fed transfer payments, strikes me as the most important and generally disregarded fiscal limit to monetary policy at the moment.

Now, this argument is not fully fleshed out. I have alluded to a decade of high real interest rates before monetary policy has an effect, in deference to the common view that the high real rates of the 1980s were in fact necessary for inflation stabilization, and caused by monetary policy. But I have not put forward models that have such dynamics. Perhaps the story that high real rates are important to monetary stabilization are wrong. Perhaps the past really was a $MV(r) = PY$ regime, and high interest rates were necessary to squeeze down M, but that in the new interest-on-reserves regime, where we are never rationed in liquidity and banks never constrained in lending, a high period of real rates will not be necessary.

But historical experience more broadly suggests that all serious monetary stabilizations are joint fiscal and monetary stabilizations, and approaching them with a large stock of nominal debt in hand is more difficult than without. The fiscal *limits* on monetary policy may loom large in the interest on reserves, high sovereign debt era.

8 Communication, anchoring, inflation targets and policy rules

The biggest problem we face in thinking about fiscal backing for inflation is the nebulousness of the present value of future surpluses $E_t \Sigma \beta^j s_{t+j}$. As in financial economics, the difficulty of independently measuring present values leads to interminable arguments whether asset prices vary in a way that accords with the present value formula. A well-designed fiscal backing regime should give a clearer communication of how much the fiscal backing is.

Likewise, if the government wants to inflate, how does it communicate a reduction in future surpluses? One can read the struggles of Japan in the last decade or two as a measure of just how difficult it is to reduce expectations of future surpluses! People buy Japanese debt expecting it to be paid off in real terms, no matter how much debt Japan racks up. The slow decline in US and Euro inflation despite low interest rates and large budget deficits poses a similar conundrum.

Though I have emphasized a very stylized "monetary policy" consisting of debt sales or interest rate targets with no fiscal response, in fact coordinated fiscal responses will make overall policy much more effective. If only the government could announce or communicate a commitment to what $E \sum_{j=1}^{\infty} \beta^j s_{t+j}$ would be, then inflation control would be much easier. Historically, people understood inflation to be more of a fiscal than a monetary phenomenon. Though I have emphasized some "monetary" control, it is clear that in the interest on reserves regime the fiscal coordination will be more important, and reviving a centuries old inquiry into appropriate fiscal policy will be necessary.

In fact, many monetary institutions can be read precisely as such commitment devices. A review of some past devices, an interpretation of inflation targeting as a current device suggests how we might construct better communication and commitment devices in the future

8.1 Gold standard, exchange rate pegs and currency boards

The gold standard is often thought of as a monetary device, a way to give notes value by backing. (Though gold itself plausibly derived most of its value by scarcity rather than frictionless industrial value.) It is, I think, better seen as a fiscal commitment.

Few governments on a gold standard backed even 100% of their note issue with gold reserves, and no government ever backed the entire face value of its debt. Furthermore, a government in fiscal trouble could be sure to grab the gold reserves and inflate newly unbacked notes.

So what happens when a gold-standard government must pay off debt that comes due, in quantity bigger than the gold reserves, or must defend against a run of partially backed note issues? It must either raise current taxes less spending, to obtain gold, or it must credibly commit to raising future taxes less spending so that it can borrow gold.

A gold standard is thus a *fiscal* commitment. It is a way of saying, "we promise to raise surpluses as necessary to pay off our debt at the price level of 1.0 relative to gold, no more and no less."

Like all promises, this one is too easily broken. The gold standard era is as much a history of crashes and runs as it is a history of centuries of happy price stability. But it at least shows the possibility of making a commitment or promise that communicates expectations of the present value of surpluses rather than think of them as a nebulous present value somewhat like the present value of stock earnings.

A currency board or foreign exchange rate peg operate in the same way. They seem to be purely monetary devices, a way to give money value by its backing rather than scarcity in exchange. (Likewise they leave unanswered where the value of the targeted money comes from.) But in fact they are fiscal commitment devices. Exchange rate pegs do not operate with 100% reserves, and even more so do not operate with 100% backing of all government nominal debt. Thus, if there is a test of the peg, or if government debt comes due, the government must raise taxes less spending, in this case raising from its citizens claims abroad that can be used to get foreign currency, now or in present value form. Currency boards operate with 100% *money* reserves, but not 100% *debt* reserves. Thus, again, the government's ability to tax its citizens when debt comes due fundamentally drives the viability of the regime. And of course the government can always grab the board's reserves if surpluses are insufficient, as Argentinians found out to their dismay.

The gold standard, currency boards, and exchange rate pegs are particularly useful when the objective is to stoke rather than to contain inflation. By cutting the gold price or devaluing the currency, the government is able to very clearly communicate that the fiscal underpinnings of monetary policy $E\sum\beta^j s_{t+j}$ are lower, in a way that fiat-currency countries such as Japan, the US, and the EU have not been able to do. It's almost as good as the clearest possible such device, a currency reform. The difference is that a devaluation is more easily renegotiated.

In sum, the gold standard, currency pegs, and currency boards are at heart fiscal *communication* devices, ways to communicate to money holders just how much surplus, and no more, will be used to redeem nominal debt B_{t-1}. They are somewhat less successful *commitment* devices, as the frequent inabilities to make good on promises in inflationary environments proves.

8.2 Inflation targeting and anchoring

It is generally thought that inflation expectations are now "anchored," allowing the government latitude for current inflation to go up and down without changes in expected future inflation, or anchoring the Phillips curve which wandered so unfortunately in the 1970s. Countries that have adopted formal inflation targets view that announcement as an important part of this "anchoring"

The question is, what does "anchoring" mean and what institutions have achieved it?

The standard Keynesian and new-Keynesian story is that the target π^* in a Taylor rule $i_t = r_t + \pi^* + \phi_\pi(\pi_t - \pi^*)$, or equivalently the intercept $\bar{\imath}$ in $i_t = r_t + \bar{\imath} + \phi_\pi \pi_t$, where r_t is a time-varying "natural rate," provides the anchoring. People believe that the central bank will react to inflation above its target with higher interest rates.

I have argued elsewhere (Cochrane (2011a, 2013)) against this view. It's a bit sensible in an old-Keynesian framework since in that framework higher interest rates lead to lower future inflation, and the coefficient ϕ_π is learnable by experience. Alas, the old-Keynesian framework fails the test of being an economic model, despite a half century search for foundations. In the new-Keynesian framework, the Taylor rule is a believed commitment by the central bank to blow up the economy, to introduce instability, to raise future inflation in response to high past inflation, as a device to get the economy to jump to one of many equilibria. I have argued this is neither credible nor theoretically successful. Furthermore, there is no experience of $\pi_t - \pi^* \neq 0$ in such a model, so the belief in ϕ_π cannot come from experience.

But none of that matters here. Here it is sufficient to point to the Ricardian footnote. In any well-specified model, we must have fiscal policy coordination. So, when you read the fiscal-coordination footnote, π^* is in the models a *fiscal commitment* just like the fiscal commitment underlying the gold standard.

This insight suggests a different interpretation of explicit and implicit inflation targeting regimes. When government and central bank agree on an inflation target, this is not just a one-way commitment, constraint, and communication device for the central bank. It is also a commitment, constraint, and communication device for the fiscal authorities. It says "we will raise taxes and lower spending as required to pay off government debt at the inflation target, no more and no less."

A 2% inflation target says that the treasury will raise surpluses as necessary to pay off debt at 2% inflation, and only 2% inflation. The central bank sets $\{B_t\}$ or a nominal interest rate, which controls expected inflation, and also allows the interest rate to respond to "natural rate shocks." The Treasury commits that if inflation gets out of control it will raise taxes, and vice versa.

Though in principle the "central bank" could control expected inflation entirely with $\{B_{t-1}\}$ variation, for any surplus path, it is surely much easier for the surplus path to be anchored, as much smaller debt variation is required. Conversely, the Treasury's commitment ensures that unexpected inflation is small, and controlling unexpected inflation helps to control expected inflation and to anchor expectations.

Figure 4 then poses a challenge to the standard stylized view of monetary influence. The standard old-Keynesian story is that short term rates rise, raising real rates, then inflation falls and nominal rates follow. (Again, new-Keynesian models require that agents learn the off-equilibrium threat implied by their Taylor rule directly by some means other than data.) Beyond the 1980 episode, the more striking feature of Figure 4 is a strong positive correlation between nominal rates and inflation, both at secular and business cycle frequencies. It looks a lot more like the Fisherian view that our frictionless models suggested, guided by a slow and declining inflation target. Our inflation target is implicit. So the government (or people's understanding of the government) slowly reduces the inflation target, the Fed follows actual inflation by slowly reducing interest rates. Long term rates remain high as the credibility of the target slowly sinks in.

I think the current circumstance of zero interest rates and slowly declining inflation represents just about perfect monetary policy, heading to the Friedman rule. Others want more inflation. This analysis suggests an endorsement of ideas such as Blanchard, Dell'Ariccia, Mauro (2010) and Woodford (2012) to announce a higher inflation target to that end, but by a quite different mechanism.

The usual story is that the higher inflation target, if believed, signals that the Fed will keep interest rates low for longer. In this analysis, such a target acts instead like a devaluation. It is a statement that the *Treasury* will work to pay back debt at inflation above the target, but not so for inflation below the target. If believed, inflation would then rise when the target rises, and nominal interest rates would quickly *rise*, not decline.

8.3 The next step

Accept for a moment this view: our institutions have already adapted to the world of interest-paying money, liquidity satiation, nearly frictionless money though sticky prices, and ultimately fiscal backing, with a combination of nominal interest rate targets and implicit or explicit inflation targets providing a communication of underlying fiscal policy. Having understood our institutions this way, how can they be made better?

The fiscal nature of the inflation target, the fiscal communication, and the fiscal commitment, could all be improved. The following are some suggestive ideas.

A fiscal Taylor rule. As part of a budget process, the government could follow a fiscal Taylor rule, promising to raise surpluses when inflation (or, better, the price level) increases, and to decrease it when inflation decreases. Though the idea of any coherent budget policy – or any budgets at all – may seem quaint to American readers, other countries are able to follow more disciplined budget processes. Switzerland, for example, has a debt limit that actually leads to surpluses as it is approached.

A CPI target A gold standard is unrealistic for many reasons. Most of all, we want the CPI to be stable, not the price of gold. Now the CPI basket itself cannot be targeted. But we can target CPI futures. Alternatively, targeting the spread between nominal and real debt accomplishes essentially the same thing: it guarantees that something nominal (nominal debt) can be exchanged for something real (indexed debt). It has a similar fiscal commitment, as the real debt cannot be inflated away without default. The maturity of the debt promise should be of the order that one thinks inflation is sticky, as one wants expected inflation to adjust to the spread between real and nominal debt not the other way around. By targeting only the *spread* between real and nominal debt, we can think about removing the Fed from the business of micromanaging the level of real rates. Alternatively, the Treasury could be in charge of this peg, as it was in charge of the gold standard, emphasizing its primary responsibility for long-run inflation.

Variable coupon debt. The Achilles heel of the gold standard and currency pegs – the Achilles heel of all government-provided money, really – is the occasional breakout of inflation when governments run in to fiscal problems and fiscal promises become fiscal defaults. Historically, inflation is always and everywhere a fiscal problem. Has there been a single case of inflation by a government running substantial and prolonged fiscal surpluses?

For this reason, there is a case against making explicit fiscal promises – when they are broken, it looks more like a default rather than a mechanical reduction in the value of equity. On this basis, for example, Sims (2002) argued against dollarizaton for Mexico. Equity is a useful buffer, and nominal debt is equity when it inflates.

On the other hand, why should the government throw noise into every private contract by inflation when it wishes to default on its debt, if explicit default is costly.

Some form of "government equity" could ride to the rescue. Shiller and Kamstra (2010) have advocated "Trills," GDP-linked debt, that would automatically rise and fall as GDP, and hence potential government surpluses, rises and falls. I am not convinced that a tight rule is necessary. If the government issued long-term debt, ideally perpetuities, and had the explicit right to adjust coupons as necessary, the same buffer could be achieved. When the government is in trouble, it can lower or temporarily eliminate coupons, without needing to inflate. When the government is back to fiscal health, say at the end of war, recession, or reform, it will want to raise coupons again, to restore its access to low-cost financing.

This mechanism is really quite similar to the suspension of convertibility that governments, especially the British, followed during wars under the classical gold standard. In a war, the government suspended convertibility to gold. Inflation was moderate. After the war, the government restored convertibility at par, giving relief to those who held nominal debt during the war, profits to those who bought the debt, and faith to those who would hold the debt during the next war. In this reading, Keynes' advice to inflate the WWI debt was contingent on the idea that there would never be another war in which the government would want to borrow and suspend convertibility. Similarly, Velde (2009) tells a "Chronicle of a deflation unforetold" in which France intentionally raised the coupon rate of its bonds, just to enhance its reputation in bond markets.

Promised defaults. To the issue of the moment, if (if!) they want to create some inflation, the

US, Japan, and Europe face a conundrum in how to do it at the zero bound, and without a peg they can devalue other than each other. Interest rates can't go below zero, few are Fisherians who think that raising nominal rates and leaving them there would result in inflation, and there is no way to undo fiscal commitments just a little bit. Government debts are, unlike currency reforms, delicately set up to form expectations that higher future surpluses will pay them off. Central banks, unable to speak softly and carry a big stick, are left with speaking loudly because they have no stick, trying various "managing expectations" and "open-mouth" operations.

Absent a move to an explicit fiscally backed inflation target with something like a fiscal Taylor rule underlying it, governments could, however, make inflationary fiscal promises. For example, they could simply promise to default on debt in the future, proportional to inflation achieved between now and then. If inflation is insufficient, they will default on a proportional amount of debt. For example, a government could say its inflation target is 3%. If inflation comes in below 3%, say 0%, it will default on 3% of its debt, i.e. each bondholder will receive 97 cents on the dollar. Now the value of government debt must reflect 3% expected inflation. People will try to sell government debt, driving up the value of goods and services until the 3% inflation is achieved.

Default is messy, of course, and this is more a conceptual exercise than a realistic proposal. Like a helicopter drop, the trick is to design an intervention that communicates the desired fiscal backing, and then commits the government to following through.

9 Monetary policy doctrines

This analysis suggests a different picture of many standard monetary policy doctrines, and policy temptations.

9.1 Things not to fear and fallacies

Along with what does work, the analysis suggests that several conventional fears are not founded.

Though already mentioned, the list naturally starts with: There is no need to fear uncontrolled inflation from large reserves, a large balance sheet, a fixed inflation target, or the absence of any liquidity rationing. *Interest-paying reserves are not inflationary.*

Conversely, one need not fear a deflation spiral either. Early in the recession following the financial crisis, many observers worried that deflation would lead to a large real rate of interest, which would lead to further deflation, and so forth, potentially without end. Such a *de*flation cannot happen without fiscal backing. A country with a 100% debt to GDP ratio that experiences a 50% cumulative deflation must double the surpluses which will retire that debt, adding 100% of GDP in net present value of surpluses. There is a top of the present-value Laffer curve somewhere – a tax rate that maximizes $E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}$, including the deleterious effects of distortionary taxation on human and physical capital investment and hence on long-term growth, left out of most Laffer curve calculations.

In this analysis there is no particular reason to *want* inflation. The Friedman rule specifies negative inflation and a zero nominal rate, and periods of slow deflation such as the late 19th century were not economic calamities. One might worry about nominally sticky wages and like zero inflation with a positive nominal rate, though after a decade or so nominal stickinesses might well disappear. But there is no reason to want even the standard 2% inflation target. A price level target, in which inflation disturbances are slowly reversed, is possible too.

The standard argument for a positive inflation target is that it gives central banks room to stimulate before hitting the zero lower bound. Such room is desirable in a new-Keynesian Taylor rule as well, so that the economy can be at the "active" equilibrium with $\phi_\pi > 1$ in $i_t = i_t^* + \phi_\pi(\pi_t - \pi^*)$. If the "natural" rate i_t^* falls to zero, then $\phi_\pi > 1$ is impossible and inflation may become indeterminate (Benhabib, Schmitt-Grohé and Uribe (2002)).

Neither consideration matters in this analysis. While versions of this kind of analysis with more liquidity and pricing frictions may point to the need for nominal rates substantially below the inflation rate for other reasons, it is not necessary here for inflation control (or deflation control) as it is in standard thinking.

9.2 Multiple instruments, multiple targets. Sterilized intervention revived.

Sterilized intervention is conventionally filed as a fallacy. If the central bank has one instrument, M in $MV(i) = PY$, (or i, which works only by controlling M), then the central bank can control one thing. In this analysis, the government as a whole has two levers, B and s, and potentially more when we add long term debt and explicit fiscal rules.

For example, consider the case of Switzerland, in 2014 fighting an exchange rate appreciation from flight-to-safety foreign currency inflows. After a zero interest rate policy did not stop the inflow, Switzerland added a maximum exchange rate, and began a form of "quantitative easing" by selling Swiss Francs and buying Euros and other foreign assets. Yet the Swiss central bank worries that low interest rates are sparking house price appreciation and a domestic lending boom.

Would it not be possible for a country in this situation to impose an exchange rate maximum *and* a higher interest rate? The conventional analysis says no: One requires increasing M and the other requires decreasing M. An interest rate rise would provoke an exchange rate appreciation.

But in this analysis, it seems possible. Simply announce a higher interest rate and an exchange rate; offer to buy and sell CHF treasuries in return for reserves that pay the higher rate, and offer to exchange CHF reserves for euro reserves at the exchange rate peg.

Now, we conventionally think that higher interest rates attract capital, until the exchange rate rises so far that expected depreciation matches the higher domestic rate. In so doing, we fix the future exchange rate, so the present one must rise to generate expected depreciation. But here, it seems that we can fix the current exchange rate and allow the expected future rate to fall in order to provide the expected depreciation. The exchange rate is a relative price. The interest rate is an intertemporal price. The central bank can fix both.

Equivalently, the exchange rate is a fiscal promise, it says how much foreign exchange the government must earn to pay off its debts. The interest rate can be manipulated by changing B without changing s, as outlined above. Two levers mean two targets are possible.

This suggestion makes some sense of history as well. Central banks have pursued the apparent "fallacy" of sterilized interventions many times. More puzzling, at least initially, and more directly, governments under the gold standard targeted interest rates as well as targeting the price of gold. This action was intended as a defense of the gold standard: when governments raised interest rates, gold flowed in. But it shows the possibility at least of targeting an exchange rate (gold) and an interest rate at the same time, at least for some period.

9.3 Policy temptations

I have described a rather positive evaluation of current monetary policy – inflation is muted, and apparently inflation targets have communicated a fiscal commitment that large sovereign debts and poor forecasts have not undone. However, I have described much of conventional central bank policy as having much less power than conventionally believed. I have also described a quite muted role for central banks: to manage the maturity and liquidity structure of government debt, and provide safe interest-bearing currency.

Central banks do not take such a limited view of their mandates or what is necessary to achieve them. Cynically, one might think of institutions that have grown to expect to be very powerful - and to be expected to be very powerful – discovering their conventional tools have very little power after all, and grabbing much larger powers in order to stay important.

A prime temptation will be, not only to try to control the balance sheet while changing interest on reserves, but to use the spread between interest on reserves and treasury rates and the size of the balance sheet as separate policy instruments. For example, a future central bank anxious to stimulate might lower interest on reserves below treasury rates, in an effort to "get the banks to lend out the reserves."

In my view, this would be a big mistake. The main point to pay interest on reserves is to gain financial stability, by creating a large narrow-banking sector and driving shadow-banking production of inside interest-paying money out of business. The second the Fed tries to pay less on reserves than is paid on other instruments, the shadow-banking system of overnight repurchase agreements, money market funds, and so forth will light up again.

As I have described, in the interest on reserves regime, the balance sheet loses its connection to lending or bank deposit creation. A central bank anxious to control things will want to reestablish control by other means. If reserves do not control lending anymore, control lending by regulation. The Fed is already deep in to the project of "macroprudential" mixture of capital regulation, loan type regulation, stress test commandments, and so forth. Telling banks how much to lend and who to lend to is a natural step.

With short term interest rates fixed, the Fed has turned to attempts to manipulate a wide variety of asset prices including long-term rates and mortgage rates. Other central banks manipulate exchange rates and stock prices. With housing on everyone's mind, monitoring housing prices for signs of "bubbles" will soon turn in to manipulating house prices through restrictions on credit flows.

None of this is, in my analysis, necessary for the central bank's traditional (since 1968) job, stabilizing inflation. And none of it is necessary for financial stability either. The Fed has swallowed the awful idea of the Dodd-Frank act that financial stability requires the Federal reserve to micro-manage bank assets, and now manage asset and house prices so that banks never lose money again, rather than simply to encourage non run-prone bank liabilities so that the financial system can survive inevitable booms and busts.

And the dangers are immense. Once the central bank is in the business of supporting particular sectors – housing – and home owners at the expense of home buyers, why not others? Cars? Farmers? Exporters?

9.4 Political economy

This set of thoughts leads to an important set of political economy questions. Among other issues, I have treated the Fed and Treasury as coordinated agencies of the same government. Officials at either agency definitely do not think in such terms.

The Fed values its independence, and Fed independence has been a cornerstone of its ability to execute monetary policy when politically unpopular. Yet, independence in a democracy comes at a price, limited powers. The Fed cannot write checks to voters, i.e. execute a helicopter drop, no matter how desired such stimulation might be, because actions like that must be taken by politically accountable agencies. One may distrust Administration and Congress, but they do face voters every four years.

The wide range of unconventional policies, manipulating specific prices, allocating credit to specific markets, telling specific banks what they can and cannot do, are actions with such clear fiscal and redistributive repercussions that Fed independence cannot long survive them. Only the excellent character and competence of Fed officials has, in my mind, kept loss of independence at bay. But we are one scandal away from that loss. Already, the Fed lost power to participate in bailouts and save specific institutions.

I have opined that a large balance sheet is fine for price level control and beneficial to financial stability. In a series of thoughtful speeches, Charles Plosser (2009,2010,2012,2013) disagrees, and argues that tightening should be accompanied by selling off the balance sheet and returning to a pre-2007 configuration with small reserves that do not pay interest. In (2010), "The Fed will need to shrink the size of its balance sheet toward pre-crisis levels and return its composition to all Treasuries."

As I read his argument, however, it is primarily about the political-economy dangers of a large balance sheet, not the economic or monetary dangers. For example, Plosser (2010) writes (p.8) "the duration of the portfolio is now exposed to a great deal more interest rate risk." That is true, but the Fed's loss is the Treasury's gain. Thus it is an issue of political negotiations between Treasury and Fed, not one having anything to do with the consolidated government budget constraint.

The political-economy focus is clearer in Plosser's consideration of mortgage-backed security purchases: (2010, p.8) "the composition of the portfolio has changed for the explicit purpose of supporting a particular sector of the economy – housing – which breaks entirely new ground. The public and market participants may believe that the Fed can and will use its purchases to pursue other sorts of credit policies than has been its practice in the past." Indeed. The Fed is a poor choice of institution to manage a sovereign hedge fund, since it has so many objectives and mandates. Plosser's (2009) idea of a new Fed-Treasury accord that puts all credit risk, credit subsidization, and credit allocation squarely in the Treasury's hands, is quite sensible. But it does not argue against an arbitrarily large balance sheet composed of short-term Treasuries.

However, Plosser also writes (2010 p. 10) that "paying IOR ties together the central bank's balance sheet and the government's budget constraint, since the interest is financed by government revenues," and "If the balance sheet is perceived as a "slack" variable for policymakers, someone will want to put it to use." (2010, p. 11). Plosser basically argues here that a large balance sheet amounts to monetization of government debt, and is therefore inflationary.

Here I disagree. Interest on reserves is not financed by government revenues. Interest on \$2.6 trillion of reserves is paid by the interest on \$2.6 trillion of Treasuries. The only potential loss to the government is the interest that will now be paid on the roughly \$80 billion of required reserves, couch change (at 5%, $400 million) in the Federal budget.

The Fed cannot maintain a large balance sheet – really anything above required reserves – without paying interest. Thus, the option of maintaining a $2.6 trillion balance sheet, not paying interest on reserves, and rebating the interest on the corresponding Treasury assets, without hyperinflation, simply does not exist. We cannot count that as a "cost."

With interest on reserves, the balance sheet is *economically,* if not politically, meaningless, so it cannot be a temptation or a slack variable. It is as if the Fed were to buy green Treasuries and to issue blue ones instead, because people like the color better. Plosser's view contradicts my assertion that with full interest on reserves, reserves and treasuries are perfect substitutes.

Plosser does not argue that the Fed *cannot* raise interest rates without shrinking the balance sheet, or that the government as a whole will lose control of the price level if it does so, which are the central economic points of my argument.

However, his warning is important. Reserves are *only* non-inflationary and they are *only* immune from temptations, if they pay the same interest as short-term Treasuries. So, I think these concerns in the end emphasize the importance of always, and perhaps by law or rule, paying full market interest on reserves. If the Fed adds discretion to pay less than market interest on reserves, and to use the spread between reserves and treasuries as a second policy variable, as Kashyap and Stein (2012) suggest, all of Plosser's warnings take hold.

There are many, larger, political economy issues involved with a large balance sheet and interest-paying reserves, which I do not mean to diminish, yet will not treat in any great detail. Congress may well see "the Fed paying interest to the big banks" especially foreign banks, and object. Congress may demand that the Fed stop "subsidizing the banks not to lend money to people and businesses." Plosser's warnings that interest on reserves may lead to a loss of Fed independence, and a continuation of the trend to discretionary policy are important. Communication of the nature and advantages of the interest on reserves regime will not necessarily be easy. Yet these worries are not inevitable to the economics of the situation. And it would be a pity if we cannot move to a better economic regime because of the political constraints of the current institutional division of responsibilities between Treasury and Federal Reserve.

9.5 A new Fed-Treasury Accord

The strong interactions between monetary and fiscal policy, as well as some peculiarities of the interest on reserves regime, suggest a new Fed-Treasury accord is in order, as Plosser (2009) recommends.

As a small example, the Fed is now buying just about exactly the same amount of long-term debt as the Treasury is issuing. Why, one thinks, can they not agree on this matter? If the Treasury Bureau of Public Debt were simply to issue short-term debt in the first place, then the Fed would not have to buy up long-term debt in exchange, and then the Fed would not have interest rate risk on its balance sheet. Yet each of Fed and Treasury takes the others' decisions as if made on another planet.

Similarly, the Fed worries about mark-to-market loss on its balance sheet and potential reduction of payments to the Treasury. But from a consolidated balance sheet point of view – my view as a taxpayer – it makes not the slightest difference which branch of the government holds interest rate risk.

So, clearly, the new Fed-Treasury accord needs to specify who is in charge of the maturity structure of government debt! That maturity structure has important implications for the timing and stabilization of inflation, as I made clear above, which puts it somewhat in the Fed's corner. But the maturity structure also has important implications for the exposure of the US Treasury to interest rate shocks. By keeping a very short maturity structure in public hands, the US is exposed to roll-over risk when interest rates rise, and has missed the opportunity to lock in amazingly cheap funding for the next 30 years, which so many corporations are doing. The Bureau of Public Debt is, as far as I can tell, completely unaware of the important role it plays in determining the state-contingent path of US deficits. But a longer maturity structure makes inflation less sensitive to surplus shocks (Cochrane (2001)), again arguing that the Fed (or the agency in charge of inflation) should control it. But last on the list, though top on the list of the Bureau of Public debt, to the extent that there are violations of the expectations hypothesis, or liquidity premiums for specific maturities, appropriate choice of the maturity structure of debt can lower overall borrowing costs. Likewise, the Fed believes in its QE operations that it can exploit violations of the Modigliani Miller theorem as well as of the expectations hypothesis, to affect the yields of specific Treasury maturities, which runs counter to a Treasury philosophy of simply issuing where rates are cheap.

As above, the appearance interest rate risk on the Fed's balance sheet doesn't cause any economic problem. The Fed's loss is the Treasury's gain. A central bank can have a negative mark-to-market value up to the value of currency outstanding. The Treasury can recapitalize the Fed if needed. But such events have great political if little economic importance, so removing interest rate risk from the Fed's balance sheet is important. Once the Fed and Treasury agree who *decides* the maturity structure in private hands, the Fed should not *hold* interest rate risk.

For both Fed and Treasury, the use of interest-rate swaps would allow a lot more flexibility. If the Fed cannot sell securities back to the Treasury, and does not want to recognize the mark-to-market loss that selling securities on the open market, with coordinated Treasury repurchases, would imply, the Fed and Treasury could engage, possibly through a third party in large fixed-for-floating swaps. Similarly, if the Treasury wants to issue in specific "cheap" maturities without thereby taking on interest rates risk, it can cheaply adjust the interest rate risk independently of the maturity structure by engaging in swap transactions.

The presence of credit risk and mortgage-backed securities on the Fed's balance sheet is more troublesome. The Fed has here taken on credit risk for the US taxpayer, and is allocating credit to a particular industry. These are properly operations of the Treasury. If the Fed-Treasury accord specifies that these are proper concerns of the Fed, at least, as plosser suggests, the Fed should swap the securities with the Treasury. Like Plosser, I favor a Federal reserve with much more restricted mandate and thus much more able to be politically independent.

But these are small matters really, compared with the big one. Fiscal and monetary policy are no longer independent. The big "Fed-Treasury coordination" that is needed is the coordination of monetary and fiscal policy. Implementing an inflation target, understood as a fiscal commitment; implementing a fiscal Taylor rule; implementing a CPI standard regime, all require detailed fiscal as well as monetary plans.

10 Concluding comments; the role of central banks

The interest on reserves regime is an attractive extension of 30 years of financial and monetary innovation. It gives us interest-paying money, the end of monetary frictions, and the foundation of a more stable financial system in which government short-term debt drives out private short term debt, much as government notes drove out banknotes in the 19th century. But this apparently small extension of our institutions challenges the core of traditional monetary theory.

Some of the questions and doctrines I have addressed: There is no need to fear inflation in this regime. We *can* have price level control with no control of "money," no rationing of liquidity, no limit on central bank balance sheets, no limit of private intermediation, and under interest rate targets, even targets that violate the Taylor principle. We can enjoy full interest on "monetary" assets, and be satiated in liquidity. The Federal Reserve has the power to target nominal interest rates in this regime, though whether it can simultaneously control the size of its balance sheet is more open to question. Fortunately the size of its balance sheet is also irrelevant. Interest-paying reserves are not inflationary. To the extent that the size of the balance sheet is correlated with stimulus or restraint, the sign is likely to change: raising interest rates via interest on reserves may come with an increase in the balance sheet. The money multiplier, the link between open market operations and lending, and velocity will all become meaningless, but without any loss of inflation control.

I have explored these issues with extremely simple models. Obviously more realistic models with more realistic pricing frictions, some idea of liquidity frictions between various classes of assets, and producing more interesting dynamics, are all in need of exploration. In particular, though I have shown how purely "monetary" policy without fiscal coordination can produce changes in the real interest rate and output, I have not produced a model with the classical sign, that to tighten the Fed first raises nominal rates, real rates rise, then output and inflation fall. Or perhaps these dynamics are not there in the real world. In the real world, monetary and fiscal policy are always coordinated – changes in expectations of $\sum \beta^j s_{t+j}$ accompany all monetary moves, discount rates loom large in the present value of future surpluses, and there is a complex and state-contingent maturity structure. Mapping to data or policy predictions needs to address all these issues.

We started with what seemed like a minor and rather technical issue – in order to raise interest rates, does the Fed need to sell off its balance sheet, or can it just raise interest on reserves and keep the huge balance sheet. We have ended up, really, at once per century redefinition of role and nature of monetary policies, and of the institutions that generate price stability and financial stability.

At issue, really, is the proper role of a central bank, the question of what monetary policy can do, what it can't do, what it should do, and what it shouldn't do. The looming reevaluation is as large as Friedman (1968), the last time our views of central banks changed so much.

As a simple example, Friedman (1968) wrote that central banks *can* determine the price level, and their primary mandate *should* be to determine the price level. Plosser (2013 p. 6) echoes this view: "..in a regime with fiat currency, only the central bank can ensure price stability. Indeed, it is the one goal that the central bank can achieve over the longer run." My analysis, following in the footsteps of Sargent and Wallace (1981), denies this claim. As Sargent and Wallace wrote, "Friedman's list of the things that monetary policy cannot permanently control may have to be expanded to include inflation" (as well as output and unemployment). Perhaps they should have said "monetary policy *alone* cannot permanently control," for the central point is that once monetary frictions recede – and even before they recede in Sargent and Wallace's analysis – monetary policy cannot control the price level without fiscal backing. And as monetary frictions vanish, like the Cheshire cat, only the fiscal backing remains.

This is not heresy. Friedman's view, though we accept it as gospel now, was novel and revolutionary at the time. In the preceding Keynesian heyday, inflation was held to come from bargaining or wages price spirals, or other somewhat mysterious forces, but not, centrally, from actions of the central bank. Monetary policy was considered extremely weak, either to do good or (Friedman's view) to do much bad either. Under the gold standard, the price level was determined by that standard, operated primarily by the Treasury, and as we have seen really part of fiscal policy. The central bank had some short term role, manipulating interest rates to manage gold flows, but nobody would have thought the central bank necessary or even primarily important for price-level determination. The US didn't even have a central bank or a monetary policy through much of the 19th century, yet we did have a price level!

As a stylized, and thus surely incorrect, history, central banks started with the Bank of England. Its primary role was to intermediate government debt, as well as to organize bondholders so that debt was more likely to be repaid. By buying government debt and issuing bank liabilities, the Bank of England lowered the sovereign's borrowing costs and provided a useful liquid asset. Its first role was thus simply to manage the liquidity and maturity structure of government debt. In the 19th century, the bank of England evolved its role as lender of last resort and liquidity firehose for banking crises. The Federal Reserve was founded to be a lender of last resort and something of a bank regulator as well. Its monetary mandate was to "provide an elastic currency," rather the opposite of controlling M to control PY.

The abandonment of the gold standard, and the move to fiat money changed all that, of course. But it was only with Friedman (1968) that our current conception of a central bank was born. And now the diffusion of interest-paying electronic money and the rapid decline in transaction and communication costs changes the landscape again.

As I have analyzed it, the role of the central bank will revert to something like what it was under the gold standard. Long-run price stability is a function of the structure of government debt, fiscal promises, and fiscal commitments. The central bank has only a short-run smoothing role, as it did under the gold standard. Like it or not, the central bank retains its role as crisis preventer, on a massive scale. I have argued that by keeping a large balance sheet and encouraging 100% backed institutions to drive out run-prone inside money, the Federal Reserve could do a lot more for financial stability than its current massive regulation and crisis lending, bailing out, and asset-price propping up activities. But like it or not, that will be a role for the central bank in the years ahead. More contentious, the central bank can use its regulatory power to micromanage the economy, channel credit, determine which and which kinds of financial businesses survive, and attempt to influence asset prices. Given that conventional monetary policy is weaker and weaker, ones view on the wisdom of this sort of thing depends on one's view whether the economy and financial system need such extensive dirigisme. I think not, but that is also an issue outside the purely monetary-policy scope of this analysis. As Plosser (2013) writes, "Assigning unachievable goals to organizations is a recipe for failure.. I fear that the public has come to expect too much from its central bank and too much from monetary policy, in particular"

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12 Appendix

12.1 Frictionless Model

(This is a somewhat simplified version of Cochrane (2005) The representative household maximizes

$$E\sum_{t=0}^{\infty}\beta^t u(c_t).$$

The household receives a constant endowment $y_t = y$. It is easier to imagine a sequence of events in each period or day, though that sequencing is not important to the model.

Each evening, the government sells a face value B_{t-1} of nominal debt due the next period. Each morning t then, the government prints up B_{t-1} new dollars to pay off the outstanding debt. Households receive the dollars, sell their endowments y for dollars and buy goods c_t for dollars. At the end of the day, they must pay lump sum taxes less transfers $P_t s_t$ in dollars. I fix the real value of net taxation. This is realistic: With standard income taxes, the nominal amount of taxes are a rate times nominal income, $P_t s_t = \tau P_t y_t$, and if the price level doubles so does the nominal amount of taxes. However, I wish to leave tax distortions out of the model. The government also sells new debt B_t at a nominal bond price Q_t. The government burns all the cash it has received. The government sets $\{B_t, s_t\}$. The household chooses $\{c_t\}$ and along the way demand for bonds and money, and the price level and asset prices clear markets.

The household period budget constraint is

$$B_{t-1} + P_t y = P_t c_t + P_t s_t + Q_t B_t + M_t$$

where M_t is money held overnight. I assume that the nominal interest rate is positive, so the household chooses zero overnight money holdings, $M_t = 0$ and thus

$$B_{t-1} + P_t y = P_t c_t + P_t s_t + Q_t B_t.$$

The household's first order condition with respect to c_t and c_{t+1} yield

$$Q_t = E_t\left[\beta\frac{u'(c_{t+1})}{u'(c_t)}\frac{P_t}{P_{t+1}}\right] = \beta E_t\left[\frac{P_t}{P_{t+1}}\right]$$

where in the second equality I have used the equilibrium condition $c_t = y$. Dividing by P_t and substituting, the money in = money out condition reads

$$\frac{B_{t-1}}{P_t} + y = c_t + s_t + \beta E_t\frac{B_t}{P_{t+1}}$$

Iterating forward,

$$\frac{B_{t-1}}{P_t} = \sum_{j=0}^{\infty}\beta^j\left(c_t - y + s_t\right) + \lim_{j\to\infty}\beta^j E_t\frac{B_{t+j}}{P_{t+j+1}}$$

I impose the usual condition that the last term is zero. This transversality condition is a condition for *household* optimality, and a constraint on household borrowing from the government. If it is positive, then the household can increase consumption over income by simply not buying so much government debt. If it were negative, then the household could roll over debt forever. If we just assume that the government borrows but never lends and the price level is positive, that condition is assured.

Then, the equilibrium condition $c_t = y$ at every date implies

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_t.$$

I took some time to derive this equation in order to emphasize that it is *not* a "intertemporal government budget constraint." It combines the household budget constraint, the household's desire not to hold money, the household optimality condition, and equilibrium in the goods market. If the household wished to die holding money, the government could print money and leave money outstanding. If the household wished to hold ever increasing amounts of government debt, the government would never have to pay its debts, and the transversality condition would not hold. If the household were a growing set of overlapping generations, that condition would not, in fact hold.

It is a (very common) mistake to read this equation as a constraint which forces the government to raise surpluses $\{s_t\}$ in response to the price level. It is instead a *valuation equation.* It describes how the equilibrium price level is determined by the nominal quantity of debt outstanding and expectations of future surpluses that will soak up that debt.

While I described a "day," and nominal debt exchanged for money and back again, that feature is clearly inessential. People can transact directly with maturing government debt, and pay taxes or buy new debt by delivering maturing government debt B_{t-1}. While this looks like a cash-in-advance economy it is not by one crucial difference: the securities market is always open, and people can hold zero cash overnight.

12.2 Sticky-Price Model

In this section I build an explicit model in which prices are set one period in advance. The sticky-price setup is a simplification of Gali (1999), which I embed in a model of fiscal price determination.

Each household derives utility from the consumption of a range of goods j. Its objective is

$$\max_{\{c_{jt}, n_t\}} E \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - n_t \right]; \; c_t = \left[\int_{j=0}^{1} c_{jt}^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}$$

subject to the constraint The households' period budget constraint is

$$B_{t-1} + \pi_t = \int_{j=0}^{1} p_{jt} c_{jt} dj + S_t + Q_t B_t$$

and a transversality condition I will describe below. The household enters the period with B_{t-1} face value of government debt, receives profits from selling goods,described below, purchases a range of goods from other households, pays nominal taxes less transfers S_t, and buys new bonds B_t at price Q_t.

Demand for varieties.

We can solve the household problem in two steps: First, find the allocation across goods c_{jt} conditional on the overall level of purchases c_t, and then find the optimal allocation across time

c_t and labor supply decision n_t. We can find the first step by the associated cost minimization problem,

$$\min_{\{c_{jt}\}} \int_{j=0}^{1} p_{jt} c_{jt} dj \text{ s.t. } c_t = \left[\int_{j=0}^{1} c_{jt}^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}$$

The first order conditions for buying good j are

$$p_{jt} = \lambda \left[\int_{j=0}^{1} c_{jt}^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{1}{\sigma-1}} c_{jt}^{-\frac{1}{\sigma}}$$

$$p_{jt} = \lambda \left(\frac{c_{jt}}{c_t} \right)^{-\frac{1}{\sigma}} \tag{24}$$

where λ is the Lagrange multiplier. Raising both sides to the $1-\sigma$ power and integrating to evaluate the multiplier, we have

$$\int p_{jt}^{1-\sigma} dj = \lambda^{1-\sigma} \left(\frac{1}{c_t} \right)^{\frac{\sigma-1}{\sigma}} \int c_{jt}^{\frac{\sigma-1}{\sigma}} dj$$

$$\left[\int p_{jt}^{1-\sigma} dj \right]^{\frac{1}{1-\sigma}} = \lambda^{-\sigma} \left(\frac{1}{c_t} \right) \left[\int c_{jt}^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}$$

$$\left\{ \left[\int p_{jt}^{1-\sigma} dj \right]^{\frac{1}{1-\sigma}} \right\}^{-\frac{1}{\sigma}} = \lambda.$$

Defining the price index

$$p_t \equiv \left[\int p_{jt}^{1-\sigma} dj \right]^{\frac{1}{1-\sigma}},$$

and substituting λ in to (24), we obtain the conditional (on c_t) demand curve for each good,

$$\frac{c_{jt}}{c_t} = \left(\frac{p_{jt}}{p} \right)^{-\sigma}. \tag{25}$$

Total expenditure is

$$\int p_{jt} c_{jt} dj = \frac{c_t}{p^{-\sigma}} \int p_{jt}^{1-\sigma} dj = \frac{c_t}{p^{-\sigma}} p_t^{1-\sigma} = p_t c_t.$$

This lovely result allows us to express the consumer's problem in terms of aggregates. Now, the consumer's problem simplifies to

$$\max_{\{c_t, n_t\}} E \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - n_t \right]$$

with budget constraint

$$B_{t-1} + \pi_t = p_t c_t + S_t + Q_t B_t$$

Production.

Each household also owns a firm, which produces only one variety of good using the household's labor, with production function

$$y_{it} = An_t$$

and facing the demand curve given by (25) from all the other households. The household earns $\pi_t = p_{it}y_{it}$. The household's problem is then

$$\max_{\{c_t, n_t, p_{it}\}} E\sum_{t=0}^{\infty}\beta^t\left[u(c_t) - n_t\right] \text{ s.t.}$$

$$B_{t-1} + p_{it}y_{it} = p_t c_t + S_t + Q_t B_t$$
$$y_{it} = An_t$$
$$\frac{y_{it}}{y_t} = \left(\frac{p_{it}}{p_t}\right)^{-\sigma}$$

I use y_t in the last equation to emphasize that each household takes the aggregate consumption = output and all the other household's pricing decisions as fixed when making its own output and consumption decisions.

Given the constraints, we can let the household choose price, quantity or labor supply. This being a "sticky price" model, I express the decision in terms of price

$$\max_{\{c_t, n_t, p_{it}\}} E\sum_{t=0}^{\infty}\beta^t\left[u(c_t) - \frac{y_t}{A}\left(\frac{p_{it}}{p_t}\right)^{-\sigma}\right] \text{ s.t.}$$

$$B_{t-1} + p_{it}y_t\left(\frac{p_{it}}{p_t}\right)^{-\sigma} = p_t c_t + S_t + Q_t B_t \tag{26}$$

Flexible prices.

In the flexible price case, the household can set its price at time t. The first order condition for p_{it} is then

$$\frac{y_t}{A}\sigma\left(\frac{p_{it}}{p}\right)^{-\sigma}\frac{1}{p_{it}} = -\lambda_t(1-\sigma)y_t\left(\frac{p_{it}}{p}\right)^{-\sigma} \tag{27}$$

where λ_t is the Lagrange multiplier on the nominal period t budget constraint (26), the value of a dollar at time t. Simplifying,

$$p_{it} = \frac{1}{A\lambda_t}\frac{\sigma}{\sigma - 1}$$

This optimal price is the same for all households, so all prices are identical, and

$$p_t = \frac{1}{A\lambda_t}\frac{\sigma}{\sigma - 1} \tag{28}$$

With all prices equal, we have $y_{it} = y_t$ and $n_t = y_t/A$.

Substituting this result in the remaining household problem, we obtain

$$\max_{\{c_t, B_t\}} E\sum_{t=0}^{\infty}\beta^t\left[u(c_t) - \frac{y_t}{A}\right]$$

$$B_{t-1} + p_t y_t = p_t c_t + S_t + Q_t B_t.$$

Now we can solve the household problem and equilibrium condition. The first order condition with respect to c_t gives

$$u'(c_t) = p_t \lambda_t = \frac{1}{A} \frac{\sigma}{\sigma - 1}. \tag{29}$$

The latter equality comes from the pricing decision (28). This is a frictionless economy, so as in our endowment economy consumption is constant with no real shocks, no matter what happens to nominal quantities.

The first order condition with respect to B_t gives

$$Q_t \lambda_t = E_t \lambda_{t+1}$$

so the bond price satisfies

$$\begin{aligned} Q_t &= E_t \left[\frac{\beta u'(c_{t+1})}{u'(c_t)} \frac{p_t}{p_{t+1}} \right] \\ &= \beta E_t \left[\frac{p_t}{p_{t+1}} \right] \end{aligned}$$

(p_t is known at time t, but this is prettier.)

Substituting, the flow budget constraint becomes

$$B_{t-1} + p_t y_t = p_t c_t + S_t + \beta E_t \left[\frac{p_t}{p_{t+1}} \right] B_t$$

Taxes. The government charges net lump-sum *real* taxes in the amount s_t so $S_t = p_t s_t$. This is not an unnatural assumption. For example, if the government charged a rate τ on nominal income $S_t = \tau p_t y_t$, then the real tax revenue would be fixed $s_t = \tau y_t$. I specify lump sum taxes to avoid dealing with distortions.

Present values. Dividing by p_t

$$\frac{B_{t-1}}{p_t} + y_t = c_t + s_t + \beta E_t \left[\frac{B_t}{p_{t+1}} \right]$$

and iterating forward,

$$\frac{B_{t-1}}{p_t} = E_t \sum_{j=1}^{k} \beta^j \left(c_{t+j} - y_{t+j} + s_{t+j} \right) + \beta E_t \left[\frac{B_{t+k}}{p_{t+k+1}} \right]$$

I impose that the limit of the term on the right hand side is zero. In the positive direction, this is a condition for *consumer* optimality. If not, the consumer could increase consumption and hence utility. In the negative direction, this is a standard no-ponzi condition preventing the consumer from borrowing larger and larger amounts. $B > 0$ and $p > 0$ – the government does not lend, and prices must be positive – serve the same purpose.

$$\frac{B_{t-1}}{p_t} = E_t \sum_{j=1}^{k} \beta^j \left(c_{t+j} - y_{t+j} + s_{t+j} \right)$$

Finally, we impose the equilibrium condition,

$$c_t = y_t.$$

This condition determines the overall price level,

$$\frac{B_{t-1}}{p_t} = E_t \sum_{j=1}^{k} \beta^j s_{t+j}$$

just as in the endowment-economy model.

Prices set one period in advance

To create a sticky price version of this model, I require that each household set its price p_{it} one period in advance. The household is committed to supply whatever demand there is at the posted price. The demand curve faced by each household producer is still

$$\frac{y_{it}}{y_t} = \left(\frac{p_{it}}{p_t}\right)^{-\sigma}.$$

so, with all prices still equal, individual demand will equal aggregate demand. But now aggregate demand y_t can vary over time.

The first order condition of the problem (26), which I repeat here,

$$\max_{\{c_t,n_t,p_{it}\}} E\sum_{t=0}^{\infty} \beta^t \left[u(c_t) - \frac{y_t}{A}\left(\frac{p_{it}}{p_t}\right)^{-\sigma}\right] \text{ s.t.}$$

$$B_{t-1} + p_{it}y_t\left(\frac{p_{it}}{p_t}\right)^{-\sigma} = p_t c_t + S_t + Q_t B_t$$

with respect to p_{it} in this case becomes, in place of (27),

$$E_{t-1}\frac{y_t}{A}\sigma\left(\frac{p_{it}}{p}\right)^{-\sigma}\frac{1}{p_{it}} = -E_{t-1}\left[\lambda_t(1-\sigma)y_t\left(\frac{p_{it}}{p}\right)^{-\sigma}\right]$$

which we simplify to

$$p_{it} = \frac{1}{AE_{t-1}(\lambda_t)}\frac{\sigma}{\sigma-1}.$$

again all prices are identical, and

$$p_t = \frac{1}{AE_{t-1}(\lambda_t)}\frac{\sigma}{\sigma-1} \tag{30}$$

Output now is $y_{it} = y_t$ and thus labor supply $n_t = y_t/A$. The household problem simplifies then to

$$\max_{\{c_t,B_t\}} E\sum_{t=0}^{\infty} \beta^t \left[u(c_t) - \frac{y_t}{A}\right]$$

$$B_{t-1} + p_t y_t = p_t c_t + S_t + Q_t B_t.$$

The first order condition with respect to c_t still gives

$$u'(c_t) = p_t \lambda_t.$$

However, the new pricing rule (30) now means (29) becomes

$$E_{t-1}\left[u'(c_t)\right] = p_t E_{t-1}(\lambda_t) = \frac{1}{A}\frac{\sigma}{\sigma-1}. \tag{31}$$

This is really the crucial difference. *Expected* marginal utility is constant. But nominal shocks will have real effects. A too low price will induce too much output, and too much consumption.

The first order condition with respect to B_t gives

$$Q_t \lambda_t = E_t \lambda_{t+1}$$

as before, and thus

$$Q_t = E_t\left[\beta \frac{u'(c_{t+1})}{u'(c_t)}\frac{p_t}{p_{t+1}}\right].$$

However, from (31), we can no longer conclude that c_t and the real interest rate are constant.

The flow budget constraint becomes

$$B_{t-1} + p_t y_t = p_t c_t + p_t s_t + E_t\left[\beta \frac{u'(c_{t+1})}{u'(c_t)}\frac{p_t}{p_{t+1}}\right] B_t$$

$$\frac{B_{t-1}}{p_t} + y_t = c_t + s_t + \beta\frac{E_t u'(c_{t+1})}{u'(c_t)}\frac{B_t}{p_{t+1}}$$

where I have used the fact that p_{t+1} is known at time t. Using (31),

$$u'(c_t)\frac{B_{t-1}}{p_t} = u'(c_t)\left[c_t - y_t + s_t\right] + \beta E_t\left[u'(c_{t+1})\right]\frac{B_t}{p_{t+1}}$$

$$u'(c_t)\frac{B_{t-1}}{p_t} = E_t \sum_{j=0}^{\infty} \beta^j u'(c_{t+j})\left[c_{t+j} - y_{t+j} + s_{t+j}\right]$$

equilibrium $c_t = y_t$ requires that p_t obey

$$u'(c_t)\frac{B_{t-1}}{p_t} = E_t \sum_{j=0}^{\infty} \beta^j\left[u'(c_{t+j})s_{t+j}\right]$$

Now, in this simple model with one-period price stickiness, we have from (31) that $E_t u'(c_{t+j}) = \frac{1}{A}\frac{\sigma}{\sigma-1}$ for $j \geq 1$. If the covariance between marginal utility and surpluses is zero, then

$$u'(c_t)\frac{B_{t-1}}{p_t} = u'(c_t)s_t + \frac{1}{A}\frac{\sigma}{\sigma-1}E_t\sum_{j=1}^{\infty}\beta^j s_{t+j}$$

$$\frac{u'(c_t)}{u'(\bar{c})}\frac{B_{t-1}}{p_t} = \frac{u'(c_t)}{u'(\bar{c})}s_t + E_t\sum_{j=1}^{\infty}\beta^j s_{t+j}$$

in this case, marginal utility $u'(c_t)$ must do all the adjusting when there is a surplus shock, as the price level cannot move.

12.3 Three equation model

This section sets out the algebra for the three equation model. For more details, see Cochrane (2011a)

The model is

$$
\begin{aligned}
y_t &= E_t y_{t+1} - \sigma\left(i_t - E_t \pi_{t+1}\right) + x_{dt} \\
\pi_t &= \beta E_t \pi_{t+1} + \gamma y_t + x_{\pi t} \\
i_t &= \phi_\pi \pi_t + x_{it}
\end{aligned}
$$

In vector form,

$$
\begin{bmatrix} y_{t+1} \\ \pi_{t+1} \\ x_{dt+1} \\ x_{\pi t+1} \\ x_{it+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{\beta}\left(\beta+\sigma\gamma\right) & -\frac{\sigma}{\beta}\left(1-\beta\phi_\pi\right) & -1 & \frac{\sigma}{\beta} & \sigma \\ -\frac{\gamma}{\beta} & \frac{1}{\beta} & 0 & -\frac{1}{\beta} & 0 \\ 0 & 0 & \rho_d & 0 & 0 \\ 0 & 0 & 0 & \rho_\pi & 0 \\ 0 & 0 & 0 & 0 & \rho_i \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \\ x_{dt} \\ x_{\pi t} \\ x_{it} \end{bmatrix} + \begin{bmatrix} \delta_{yt+1} \\ \delta_{\pi t+1} \\ \varepsilon_{dt+1} \\ \varepsilon_{\pi t+1} \\ \varepsilon_{it+1} \end{bmatrix}
$$

I proceed analytically. (I used Scientific Workplace's symbolic math toolkit) The eigenvalues are

$$
\lambda = \frac{1}{2\beta}\left(1+\beta+\sigma\gamma \pm \sqrt{\left(1+\beta+\sigma\gamma\right)^2 - 4\beta\left(1+\sigma\gamma\phi_\pi\right)}\right), \rho_d, \rho_\pi, \rho_i
$$

The eigenvectors of the unstable eigenvalues are

$$
\begin{bmatrix} 1-\beta-\sigma\gamma+\sqrt{\left(1+\beta+\sigma\gamma\right)^2 - 4\beta\left(1+\sigma\gamma\phi_\pi\right)} \\ 2\gamma \\ 0 \\ 0 \\ 0 \end{bmatrix} \leftrightarrow \lambda_-,
$$

$$
\begin{bmatrix} 1-\beta-\sigma\gamma-\sqrt{\left(1+\beta+\sigma\gamma\right)^2 - 4\beta\left(1+\sigma\gamma\phi_\pi\right)} \\ 2\gamma \\ 0 \\ 0 \\ 0 \end{bmatrix} \leftrightarrow \lambda_+
$$

The eigenvectors of the stable eigenvalues are

$$
\begin{bmatrix} 1-\rho_d\beta \\ \gamma \\ \left(1-\rho_d\right)\left(1-\beta\rho_d\right)+\sigma\gamma\left(\phi_\pi-\rho_d\right) \\ 0 \\ 0 \end{bmatrix} \leftrightarrow \rho_d
$$

$$
\begin{bmatrix} \sigma\left(\rho_\pi-\phi_\pi\right) \\ 1-\rho_\pi \\ 0 \\ \left(1-\rho_\pi\right)\left(1-\beta\rho_\pi\right)+\sigma\gamma\left(\phi_\pi-\rho_\pi\right) \\ 0 \end{bmatrix} \leftrightarrow \rho_\pi
$$

$$\begin{bmatrix} -\sigma\left(1-\rho_i\beta\right) \\ -\sigma\gamma \\ 0 \\ 0 \\ \left(1-\rho_i\right)\left(1-\rho_i\beta\right)+\sigma\gamma\left(\phi_\pi-\rho_i\right) \end{bmatrix} \leftrightarrow \rho_i$$

The model dynamics are then

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} 1-\rho_d\beta & \sigma\left(\rho_\pi-\phi_{\pi,0}\right) & -\sigma\left(1-\rho_i\beta\right) \\ \gamma & 1-\rho_\pi & -\sigma\gamma \end{bmatrix} \begin{bmatrix} z_{dt} \\ z_{\pi t} \\ z_{it} \end{bmatrix}$$

$$\begin{bmatrix} z_{dt} \\ z_{\pi t} \\ z_{it} \end{bmatrix} = \begin{bmatrix} \rho_d & 0 & 0 \\ 0 & \rho_\pi & 0 \\ 0 & 0 & \rho_i \end{bmatrix} \begin{bmatrix} z_{dt-1} \\ z_{\pi t-1} \\ z_{it-1} \end{bmatrix} + \begin{bmatrix} v_{dt} \\ v_{\pi t} \\ v_{it} \end{bmatrix}$$

where the z and the x are related by

$$\begin{aligned} x_{dt} &= \left[\left(1-\rho_d\right)\left(1-\rho_d\beta\right)+\sigma\gamma\left(\phi_\pi-\rho_d\right)\right] z_{dt} \\ x_{\pi t} &= \left[\left(1-\rho_\pi\right)\left(1-\rho_\pi\beta\right)+\sigma\gamma\left(\phi_\pi-\rho_\pi\right)\right] z_{\pi t} \\ x_{it} &= \left[\left(1-\rho_i\right)\left(1-\rho_i\beta\right)+\sigma\gamma\left(\phi_\pi-\rho_i\right)\right] z_{it} \end{aligned}$$

Similarly the shocks v are related to fundamental shocks x by

$$\begin{aligned} \varepsilon_{dt} &= \left[\left(1-\rho_d\right)\left(1-\rho_d\beta\right)+\sigma\gamma\left(\phi_\pi-\rho_d\right)\right] v_{dt} \\ \varepsilon_{\pi t} &= \left[\left(1-\rho_\pi\right)\left(1-\rho_\pi\beta\right)+\sigma\gamma\left(\phi_\pi-\rho_\pi\right)\right] v_{\pi t} \\ \varepsilon_{it} &= \left[\left(1-\rho_i\right)\left(1-\rho_i\beta\right)+\sigma\gamma\left(\phi_\pi-\rho_i\right)\right] v_{it} \end{aligned}$$

It's interesting to carry along the i response. From

$$i_t = \phi_\pi \pi_t + x_{it},$$

we can simply append the i to the response variables as

$$\begin{bmatrix} y_t \\ \pi_t \\ i_t \end{bmatrix} = \begin{bmatrix} 1-\rho_d\beta & \sigma\left(\rho_\pi-\phi_{\pi,0}\right) & -\sigma\left(1-\rho_i\beta\right) \\ \gamma & 1-\rho_\pi & -\sigma\gamma \\ \gamma\phi_\pi & \left(1-\rho_\pi\right)\phi_\pi & -\sigma\gamma\rho_i+\left(1-\rho_i\right)\left(1-\rho_i\beta\right) \end{bmatrix} \begin{bmatrix} z_{dt} \\ z_{\pi t} \\ z_{it} \end{bmatrix}$$

For the money impulse response function, we only need the last column. In the end, we just simulate

$$\begin{bmatrix} y_t \\ \pi_t \\ i_t \end{bmatrix} = \begin{bmatrix} -\sigma\left(1-\rho_i\beta\right) \\ -\sigma\gamma \\ -\sigma\gamma\rho_i+\left(1-\rho_i\right)\left(1-\rho_i\beta\right) \end{bmatrix} \begin{bmatrix} z_{it} \end{bmatrix}$$

$$\begin{aligned} z_{it} &= \rho z_{it-1} + v_{it} \\ v_{it} &= \frac{\varepsilon_{it}}{\left(1-\rho_i\right)\left(1-\rho_i\beta\right)+\sigma\gamma\left(\phi_\pi-\rho_i\right)} \end{aligned}$$